Partial Correlations

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Mplus Web Note No.26
Version 1

June 10, 2025

1 Introduction

A new diagnostic tool is available in Mplus 8.12 based on the partial correlation between variables. Suppose that V is the vector of all observed variables in the model: endogenous and exogenous. Denote by V_i the i-th variable in the vector V. The partial correlation between V_i and V_j is defined as

$$\rho_{ij} = Corr(V_i, V_j | V_{-(i,j)}), \tag{1}$$

where $V_{-(i,j)}$ is the vector of all variables V without V_i and V_j. The partial correlation is the correlation between a pair of variables conditional on all other variables in the model. Partial correlations have a prominent role in Network analysis, see Epskamp and Fried (2018), but here we discuss how partial correlations can be used for SEM diagnostics. The diagnostic is based on comparing the partial correlations $\rho_{ij}(H_0)$ of the structural model H_0 and the partial correlations $\rho_{ij}(H_1)$ of the unrestricted variance covariance model H_1. That is, the partial correlations can be computed using model H_0 and using model H_1. Model H_1 essentially uses the sample covariance matrix to compute the partial correlations. However, if the data contains missing values, the H1 model uses the EM-algorithm estimate for the covariance matrix. Similarly, if the data includes categorical variables, the H1 model uses the polychoric covariance/correlation matrix to compute the partial correlations. For brevity, we refer to $\rho_{ij}(H_1)$ as the sample partial correlations.

If the H_0 model is correct and represents well the unrestricted covariance matrix of H_1, the two partial correlations will be similar. A large discrepancy between the two partial correlations indicates a model misfit. Mplus reports the residuals of the partial correlations

$$\rho_{ij}(H_1) - \rho_{ij}(H_0) \tag{2}$$

in the RESIDUAL output, which is obtained with the option OUTPUT: RESIDUAL.

Typically in SEM, a chi-square tests H_0 against the null hypothesis of H_1. The chi-square is a global test of fit, i.e., an overall measure of fit. The residual of the partial correlation is a localized measure of fit. Currently Mplus does not compute a standard error for (2) or a significance test. However, because it is on a standardized correlation scale, the residuals are directly comparable. This way we can identify a specific part of the data that is most misfitted by the model.

The residual of the partial correlation is similar to the residual of the (unconditional/marginal) correlation that is also reported in Mplus RESIDUAL output. However, the partial correlation is more directly connected to potential model parameters. For example, in a factor analysis model, the estimated correlation is the combination of the factor analysis model and the correlated uniqueness (if it is estimated in the model). In contrast, the estimated partial correlation is much more directly related to the correlated uniqueness because it is concerned only with the relationship between V_i and V_j and not with $V_{-(i,j)}$ or the relationship between V_i, V_j and $V_{-(i,j)}$ (which are the basis for the factor model). Thus we expect that the residuals of the partial correlations would be more useful in detecting model misfit and the need for correlated uniqueness.

Another important feature of the partial correlation is that it is not directly connected to the unconditional correlation. The information that one of the two correlations is positive, negative or zero does not predict the status of the other correlation.

2 Latent Variables and Network Analysis

Consider a factor analysis model with 1 factor and P indicators all with the same size loadings and error variance. Unconditional correlation does not change as P increases. However, as P increases, the partial correlation between the variables goes to zero. This is because as P increases, the number of variables $P-2$ that we condition on also increases. In turn, this reduces the conditional correlation as a bigger part is explained by the $P-2$ variables.

A factor analysis model postulates that conditional on the factor there is no residual correlation (conditional independence). If we had included the factor in the variable that we condition on in (1), then the partial correlation would be zero. However, even without including the factor, the same thing nearly happens. When P increases, the $P-2$ variables we condition on are sufficient to measure the factor almost precisely (as P increases). Thus, conditioning on the $P-2$ variables is approximately the same as conditioning also on the factor. This also shows that in a factor analysis model as P increases, all partial correlations converge to 0. The same logic applies to other models with large numbers of variables. The more variables there are, the more likely it is that all partial correlations will be small. As long as the model postulates that the variable correlation can be explained by modeling features that are not specific to the pair of variables but are model-wide features, the partial correlation will be small. If the model postulates conditional independence between variables as in the factor analysis model, the estimated partial correlations are likely to be small. Modeling components such as latent factors are aimed at explaining correlations between all the variables in the model and not local features such as partial correlations. Partial correlations are likely to be affected by model parameters involving just the pair of variables. Such parameters are residual covariance or a regression parameter between the two variables.

The above logic applies mostly to larger models, i.e, models with larger numbers of indicators. Factor models with 2, 3, 4, or 5 indicators will not fall precisely in the above description. Epskamp and Fried (2018) discuss relationships between latent variables and clusters in partial correlation network analysis (based on $\rho_{ij}(H_1)$). They suggest that a cluster of connected variables may be indicative of a factor analysis. We argue here that this mostly applies to a small number of variables. For larger numbers of variables as illustrated above, the partial correlations will be very small and would not represent a cluster of connected variables in a partial correlation network.

3 Correlated Uniqueness

In this section we illustrate how the residuals of the partial correlations can be used to detect the need for correlated uniqueness parameters. Consider as an example a factor analysis model with 1 factor and 10 indicators which also includes three residual covariances. Data is generated as in Figure 1. The three residual covariances are for the following pairs of variables (y_3, y_4), (y_9, y_{10}), and (y_2, y_7). Next, we estimate the factor analysis model without any correlated uniqueness, see Figure 2. Here we request the RESIDUAL output. The residuals for the partial correlations output is given in Figure 3. The top 5 partial correlations by absolute values are reported in Table 1. We see that the top 3 coincide with those used in the data generation. For comparison, we also include in Table 1 the residuals of the corresponding marginal correlations. We see here that the discrepancy in the marginal correlations appears smaller than that in the partial correlations. Interestingly, the reduction in the residual appears to be uniform (divided by half). An approximate formula can be constructed to explain this proportionality. Suppose that $D(V_i, V_j)$ is the residual for the partial correlation of V_i and V_j given in (2), and $d(V_i, V_j)$ is the residual of the marginal correlation. Suppose that $R^2_{V_i}$ and $R^2_{V_j}$ are the R^2 values representing the proportion of variance explained by the factor model for the two variables. Then

$$d(V_i, V_j) \approx D(V_i, V_j)\sqrt{(1 - R^2_{V_i})(1 - R^2_{V_j})}. \tag{3}$$

Because the R^2 values are approximately 1/2 in out example, the residuals of the marginal correlations are approximately half of the residuals for the partial correlations. Higher R^2 values will lead to greater reduction for the residual for the marginal correlation. This approximate formula will break down in more complex situations, however.

We conclude that the residuals of the partial correlations are an efficient way to detect needed residual covariance parameters among the items.

Figure 1: Generating factor analysis data with correlated uniqueness

```
montecarlo:
        names = y1-y10;
        nobs = 1000;
        nreps = 1;
        save = 1.dat;

model montecarlo:
        f by y1@1 y2-y10*1;
        y1-y10*1;
        f*1;
        y3 with y4*0.5;
        y9 with y10*0.4;
        y2 with y7*0.3;
```

Figure 2: Estimating a factor model

```
variable: names = y1-y10;
data: file = 1.dat;
model: f by y1-y10;
output: residual;
```

Figure 3: Residuals for partial correlations in factor analysis

Residuals for Partial Correlations

	Y1	Y2	Y3	Y4	Y5
Y1	0.000				
Y2	0.038	0.000			
Y3	-0.052	-0.029	0.000		
Y4	-0.046	-0.120	0.425	0.000	
Y5	-0.003	0.036	-0.026	-0.020	0.000
Y6	0.044	-0.089	-0.058	-0.045	0.061
Y7	0.014	0.305	-0.134	0.015	-0.058
Y8	0.042	-0.089	-0.001	-0.047	0.051
Y9	-0.053	0.002	-0.067	-0.074	0.007
Y10	-0.010	-0.011	-0.052	-0.065	-0.089

Residuals for Partial Correlations

	Y6	Y7	Y8	Y9	Y10
Y6	0.000				
Y7	0.016	0.000			
Y8	0.044	0.043	0.000		
Y9	-0.022	-0.093	-0.009	0.000	
Y10	0.040	-0.093	-0.053	0.358	0.000

Table 1: Top 5 by absolute value residuals for partial correlations in factor analysis model

Variables	Residual for Partial Correlation	Residual for Correlation
(Y_3, Y_4)	0.425	0.188
(Y_9, Y_{10})	0.358	0.177
(Y_2, Y_7)	0.305	0.143
(Y_3, Y_7)	-0.134	-0.055
(Y_2, Y_4)	-0.120	-0.048

4 Direct effects

In this section we illustrate how the residuals of the partial correlations can be used to detect direct effects in a MIMIC model. We use a model with 10 indicators, 1 factor, and 5 covariates. Figure 4 shows the input file for the data generation. We include in the data generating model 3 direct effects from X_1 to Y_1, from X_4 to Y_3, and from X_3 to Y_7. We analyze the data using the MIMIC model without any direct effects, see Figure 5. The residuals of the partial correlations are reported in Figure 4. Table 2 contains the top 5 values. We see here again that the partial correlations perfectly identified the candidates for direct effect model modification. The residuals for the corresponding marginal correlations appear smaller in this example as well. The approximate reduction formula (3) appears to hold here as well.

Table 2: Top 5 by absolute value residuals for partial correlations in MIMIC model

Variables	Residual for Partial Correlation	Residual for Correlation
(Y_7, X_3)	0.464	0.291
(Y_3, X_4)	0.341	0.196
(Y_1, X_1)	0.251	0.156
(Y_2, X_3)	-0.089	-0.046
(Y_5, X_1)	-0.088	-0.050

Figure 4: Generating MIMIC data with direct effects

```
montecarlo:
        names = y1-y10 x1-x5;
        nobs = 1000;
        nreps = 1;
        save = 1.dat;

model montecarlo:
        f by y1@1 y2-y10*1;
        y1-y10*1; x1-x5*1;
        f*1;
        f on x1*0.1 x2*0.2 x3*0.3 x4*0.4 x5*0.5;
        y3 on x4*0.4;
        y1 on x1*0.3;
        y7 on x3*0.6;
```

Figure 5: Estimating a MIMIC model

```
variable: names = y1-y10 x1-x5;

data: file = 1.dat;

model: f by y1-y10; f on x1-x5;

output: residual;
```

Figure 6: Residuals for partial correlations in MIMIC model

Residuals for Partial Correlations

	Y1	Y2	Y3	Y4	Y5
Y1	0.000				
Y2	-0.041	0.000			
Y3	0.039	0.077	0.000		
Y4	-0.003	0.016	-0.043	0.000	
Y5	0.002	0.007	-0.018	0.026	0.000
Y6	-0.050	0.029	-0.032	-0.042	0.006
Y7	-0.009	0.010	-0.022	0.044	0.015
Y8	-0.008	-0.051	-0.019	0.006	0.003
Y9	0.030	0.047	-0.032	-0.011	-0.033
Y10	0.012	-0.040	0.011	0.011	0.017
X1	0.251	0.033	-0.010	-0.063	-0.088
X2	0.012	-0.044	0.018	0.024	0.050
X3	0.006	-0.089	-0.070	-0.077	-0.078
X4	-0.045	-0.061	0.341	-0.025	-0.063
X5	0.074	-0.085	0.014	0.006	-0.010

Residuals for Partial Correlations

	Y6	Y7	Y8	Y9	Y10
Y6	0.000				
Y7	0.051	0.000			
Y8	-0.030	0.013	0.000		
Y9	0.054	-0.053	0.042	0.000	
Y10	-0.008	-0.067	0.072	-0.003	0.000
X1	-0.003	-0.012	-0.071	-0.046	0.022
X2	0.034	0.014	-0.048	-0.018	-0.042
X3	-0.015	0.464	-0.016	-0.083	-0.034
X4	0.001	-0.033	-0.028	-0.078	-0.033
X5	-0.015	0.067	-0.057	0.006	-0.000

Residuals for Partial Correlations

	X1	X2	X3	X4	X5
X1	0.000				
X2	0.001	0.000			
X3	0.001	-0.004	0.000		
X4	0.005	0.001	0.034	0.000	
X5	-0.018	-0.007	-0.021	0.002	0.000

5 Conclusion

The residuals of the partial correlations are uniquely suited to identify local model deficiency that can be addressed with parameters specific to a pair of variables, such as residual covariances and direct effects. The method can be used with continuous and categorical outcomes. With categorical outcomes, Mplus utilizes the polychoric correlation matrix to construct partial correlations.

References

[1] Epskamp, S. & Fried, E. (2018) A tutorial on regularized partial correlation networks. Psychological Methods, 23, 617–634.