

5.2 EFA longitudinal alignment

Outline

- EFA background
- EFA variations
 - ESEM, PSEM
 - Second-order, SEFA
 - Bi-factor, DSEFA
 - Target
- EFA in an SEM setting
 - MIMIC
 - EFA/CFA on EFA/CFA
- EFA in a multiple-group setting
 - EFA alignment
- EFA in a longitudinal setting
 - EFA longitudinal invariance testing
 - **EFA longitudinal alignment**
 - EFA growth modeling
- Further topics
- EFA theory

Slide 115 returns to the Outline, showing that we are still in the longitudinal setting. The next longitudinal topic is alignment.

Longitudinal Alignment

- Section 5.3 of Asparouhov & Muthén (2024). Penalized structural equation models. *Structural Equation Modeling: A Multidisciplinary Journal*, 31(3), 429–454.
 - 1 factor at several timepoints using the ALIGNMENT option
- PSEM can be used with DIFF priors for factor loadings so that factor variances can be estimated for $t > 1$
 - See the Asparouhov (2023) Mplus Workshop slides 52 - <https://www.statmodel.com/download/M3TeachingSlides.pdf>
- Automated longitudinal alignment for EFA using the ALIGNMENT option

Slide 116 turns to longitudinal alignment. The alignment approach avoids the problem of ill-fitting metric and scalar invariance over time, relying only on configural invariance.

The slide shows 3 background sources for longitudinal alignment. The 2024 PSEM paper has a section 5.3 giving an example of alignment with 1 factor measured at several timepoints. Clearly, this does not cover the EFA context.

PSEM can also be used with DIFF priors that hold factor loadings approximately equal over time. In this way, factor variance can be estimated for all but the first timepoint. This procedure is described in the 2023 workshop slides by Asparouhov.

Finally, we have the automated approach for EFA that will be described here.

Input for Longitudinal Alignment of PA Factors

```
USEVARIABLES = relax1-excit7;
! 6 PA items, 3 low arousal, 3 high arousal:
! relaxed (pala1) satisfied (pala2) confident (pala3)
! happy (paha1) energetic (paha2) excited (paha3)

ANALYSIS: ESTIMATOR = MLR;
ALIGNMENT = FIXED; ! this is the only change
! to the measurement invariance input that used
! MODEL = CONFIGURAL etc

MODEL: ! random intercepts for all 6 items:
i1 BY relax1-relax7@1;
i2 BY satis1-satis7@1;
i3 BY conf1-conf7@1;
i4 BY happy1-happy7@1;
i5 BY energ1-energ7@1;
i6 BY excit1-excit7@1;
! auto-regressions among factor indicators residuals:
relax2^-relax7^ PON relax1^-relax6^ (ar1);
satis2^-satis7^ PON satis1^-satis6^ (ar2);
conf2^-conf7^ PON conf1^-conf6^ (ar3);
happy2^-happy7^ PON happy1^-happy6^ (ar4);
energ2^-energ7^ PON energ1^-energ6^ (ar5);
excit2^-excit7^ PON excit1^-excit6^ (ar6);
```

Slides 117-118 show the input for the automated alignment approach. As the comment in the ANALYSIS command says, only 1 line needs to be added to the longitudinal invariance input showed earlier, saying **ALIGNMENT = FIXED**.

The input continues on the next slide.

Input for Longitudinal Alignment Cont'd

! AR1 regressions among factors
! to reduce the number of parameters:

f21-f22 ON f11-f12;
f31-f32 ON f21-f22;
f41-f42 ON f31-f32;
f51-f52 ON f41-f42;
f61-f62 ON f51-f52;
f71-f72 ON f61-f62;

i1-i6 with f11-f72@0;

! 2-factor ESEM for each time point:

MODEL t1: f11-f12 by relax1 satis1 conf1 happy1 energ1 excit1(*1);
MODEL t2: f21-f22 by relax2 satis2 conf2 happy2 energ2 excit2(*2);
MODEL t3: f31-f32 by relax3 satis3 conf3 happy3 energ3 excit3(*3);
MODEL t4: f41-f42 by relax4 satis4 conf4 happy4 energ4 excit4(*4);
MODEL t5: f51-f52 by relax5 satis5 conf5 happy5 energ5 excit5(*5);
MODEL t6: f61-f62 by relax6 satis6 conf6 happy6 energ6 excit6(*6);
MODEL t7: f71-f72 by relax7 satis7 conf7 happy7 energ7 excit7(*7);

Longitudinal Alignment Results: Measurement Part

APPROXIMATE MEASUREMENT INVARIANCE
(NONINVARIANCE) FOR TIMES

Intercepts/Thresholds							
RELAX1	1	2	3	4	5	6	7
SATIS1	1	2	3	4	5	6	7
CONF1	1	2	3	4	5	6	7
HAPPY1	1	2	3	4	5	6	7
ENERG1	1	2	3	4	5	6	7
EXCIT1	1	2	3	4	5	6	7
Loadings for F11							
RELAX1	1	2	3	4	5	6	7
SATIS1	1	2	3	4	5	6	7
CONF1	1	2	3	4	5	6	7
HAPPY1	1	2	3	4	5	6	7
ENERG1	1	2	3	4	5	6	7
EXCIT1	1	2	3	4	5	6	7
Loadings for F12							
RELAX1	1	2	3	4	5	6	7
SATIS1	1	2	3	4	5	6	7
CONF1	1	2	3	4	5	6	7
HAPPY1	1	2	3	4	5	6	7
ENERG1	1	2	3	4	5	6	7
EXCIT1	1	2	3	4	5	6	7

- No parentheses means no measurement noninvariance: All parameters are deemed invariant (scalar invariance)

Slide 119 shows the results of the alignment for the measurement part of the model. The rows represent the measurement parameters. They are intercepts for the 6 indicators and 6 loadings for each of the two EFA factors. The columns represent the 7 timepoints. Entries with parentheses are non-invariant. Because no entry has a parenthesis, all measurement parameters are invariant as judged by alignment.

Longitudinal Alignment Results: Factor Means

FACTOR INTERCEPT COMPARISON AT THE
5% SIGNIFICANCE LEVEL IN DESCENDING ORDER

Results for Factor F11

Factor Ranking	Time	Factor Intercept	Times With Significantly Smaller Factor Intercept
1	5	0.299	1 3 7
2	4	0.002	
3	1	0.000	7
4	2	-0.114	
5	6	-0.137	
6	3	-0.152	
7	7	-0.336	

Results for Factor F12

Factor Ranking	Time	Factor Intercept	Times With Significantly Smaller Factor Intercept
1	5	0.300	1 3 6
2	4	0.236	1 3 6
3	7	0.095	6
4	1	0.000	6
5	2	-0.046	
6	3	-0.055	
7	6	-0.302	

Slide 120 shows the results of the alignment for the factor means. The top part of the table shows the means for the F11 factor. The means are listed from largest to smallest. It is seen that time 5 has the largest mean and that it is significantly larger than the means at times 1, 3, and 7. The factor mean at time 1 is fixed at zero as a result of using the FIXED option.