5.1 EFA longitudinal invariance testing

Outline

Slide 107 returns to the Outline. We will now turn to invariance testing for longitudinal EFA.

Input for Longitudinal Measurement Invariance Testing

```
              USEVARIABLES = relax1-excit7;
              ! 6 PA items, 3 low arousal, 3 high arousal:
              ! relaxed (pala1) satisfied (pala2) confident (pala3)
              ! happy (paha1) energetic (paha2) excited (paha3)
              ! 7 time points

ANALYSIS:     ESTIMATOR = MLR;
              MODEL = CONFIGURAL METRIC SCALAR;

MODEL:        ! random intercepts for all 6 items:
              i1 BY relax1-relax7@1;
              i2 BY satis1-satis7@1;
              i3 BY conf1-conf7@1;
              i4 BY happy1-happy7@1;
              i5 BY energ1-energ7@1;
              i6 BY excit1-excit7@1;
              ! auto-regressions among factor indicators residuals:
              relax2^-relax7^ PON relax1^-relax6^ (ar1);
              satis2^-satis7^ PON satis1^-satis6^ (ar2);
              conf2^conf7^ PON conf1^-conf6^ (ar3);
              happy2^-happy7^ PON happy1^-happy6^ (ar4);
              energ2^-energ7^ PON energ1^-energ6^ (ar5);
              excit2^-excit7^ PON excit1^-excit6^ (ar6);
```


Slide 108 shows the input for longitudinal measurement invariance testing with EFA. This is using the PA example presented earlier.

The ANALYSIS command uses the MODEL option to test configural, metric, and scalar invariance. The rest of the specification is as shown earlier with random intercepts, and auto-regressions among factor indicator residuals.

The input continues on the next slide.

Input for Longitudinal Measurement Invariance Testing Continued: The Use of Model t

```
             ! AR1 regressions among factors
             ! to reduce the number of parameters:
             f21-f22 ON f11-f12;
             f31-f32 ON f21-f22;
             f41-f42 ON f31-f32;
             f51-f52 ON f41-f42;
             f61-f62 ON f51-f52;
             f71-f72 ON f61-f62;

             i1-i6 with f11-f72@0;

             ! 2-factor ESEM for each time point:
MODEL t1:    f11-f12 by relax1 satis1 conf1 happy1 energ1 excit1(*1);
MODEL t2:    f21-f22 by relax2 satis2 conf2 happy2 energ2 excit2(*2);
MODEL t3:    f31-f32 by relax3 satis3 conf3 happy3 energ3 excit3(*3);
MODEL t4:    f41-f42 by relax4 satis4 conf4 happy4 energ4 excit4(*4);
MODEL t5:    f51-f52 by relax5 satis5 conf5 happy5 energ5 excit5(*5);
MODEL t6:    f61-f62 by relax6 satis6 conf6 happy6 energ6 excit6(*6);
MODEL t7:    f71-f72 by relax7 satis7 conf7 happy7 energ7 excit7(*7);
```


Slide 109 continues the input with the AR1 regressions among the factors.

The bottom part of the slide shows the new feature that can be used with invariance testing. Here, an EFA is set up by ESEM for each timepoint using the MODEL t feature. This creates 7 EFA blocks, one for each timepoint. Invariance testing for these 7 EFA blocks is carried out in combination with using the ANALYSIS option model = configural metric scalar.

Longitudinal Measurement Invariance Test Results

Model	Number of Parameters	χ^2	Degrees of Freedom	P-Value
Configural	212	1246.811	733	0.0000
Metric	164	1281.246	781	0.0000
Scalar	140	1326.299	805	0.0000
Models Compared		χ^2	Degrees of Freedom	P-Value
Metric against Configural		57.341	48	0.1673
Scalar against Configural		94.980	72	0.0362
Scalar against Metric		46.660	24	0.0037

- The scalar model is typically rejected but is needed for growth modeling
- Ways out of this dilemma include:
  - Longitudinal alignment
  - Approximate invariance using PSEM or BSEM, e.g. for only the scalar part (intercepts)

Slide 110 shows the results of the measurement invariance testing. The top part of the table shows that all 3 models are rejected by chi-square. The bottom part suggests that the metric model does not fit significantly worse than the configural model, whereas the scalar models fits borderline worse than the metric model.

The scalar model is often rejected and such rejection is an obstacle to subsequent growth modeling where means are compared over time.

There are ways out of the dilemma of scalar misfit. Alignment is a key technique that will be discussed next. It is also possible to use PSEM, or the Bayesian counterpart BSEM, for approximate scalar invariance with priors for differences in the intercepts.

Automated Invariance Testing Advantages

- Also handles categorical outcomes using WLSMV with Delta and Theta parameterizations
- Automatically uses the scaling correction factors for chi-square difference testing with MLR and uses DIFFTEST with WLSMV

Slide 111 points out advantages of the automated measurement invariance testing using MODEL = configural metric scalar in combination with MODEL t.

One advantage is the handling of categorical outcomes with the WLSMV estimator using both Delta and Theta parameterization. The other advantage is the automation of scaling corrections with chi-square difference testing for MLR and difference testing using DIFFTEST with WLSMV.

Input for Longitudinal Measurement Invariance Testing with Categorical Outcomes: WLSMV

- Three 8-category ordinal items measuring 1 factor at 7 time points

```
          USEVARIABLES = bkThin1f bkThin1s bkThin2s
          bkThin3s bkThin4s bkThin5s bkThin6s
          harmO1f harmO1s harmO2s harmO3s harmO4s harmO5s
          harmO6s
          takeP1f takeP1s takeP2s takeP3s takeP4s TakeP5s takeP6s ;

          CATEGORICAL = bkThin1f - takeP6s;
          MISSING = ALL (999);

ANALYSIS: ESTIMATOR = WLSMV;
          MODEL = CONFIGURAL SCALAR;
```


Slide 112 shows an example of invariance testing with categorical outcomes using the WSLMV estimator. The example has 3 categorical indicators of a factor measured at 7 timepoints. The ANALYSIS command specifies WLSMV estimation and invariance testing.

The input continues on the next slide.

Input Continued

```
MODEL:        ! As before, but optional

MODEL t1:     f1 BY bkthin1f
              harmo1f
              takeP1f ;
MODEL t2:     f2 BY bkthin1s
              harmo1s
              takeP1s ;
MODEL t3:     f3 BY bkthin2s
              harmo2s
              takeP2s ;
MODEL t4:     f4 BY bkthin3s
              harmo3s
              takeP3s ;
MODEL t5:     f5 BY bkthin4s
              harmo4s
              takeP4s ;
MODEL t6:     f6 BY bkthin5s
              harmo5s
              takeP5s;
MODEL t7:     f7 BY bkthin6s
              harmo6s
              takeP6s ;
```


Slide 113 continues the input with the MODEL command which uses the MODEL t approach for the 7 timepoints. Because there is only 1 factor, EFA and ESEM are not relevant and the specification is done in CFA style.

Different Number of Categories for Different Time Points

- Adding one more time point leads to an error:
  - *** ERROR in MODEL command MODEL T1 and MODEL T8 are not equivalent. The categorical indicators in the same position for factors across time must have the same number of categories. Problem with: BKTHIN1F and BKTHIN7S
- With ML and Bayes, this can be handled by the * approach:
  - CATEGORICAL = bkThin1f - takeP7s(*);
- WLSMV cannot handle the * approach
- ML can handle it but requires numerical integration and there are typically too many dimensions of integration due to many factors
- Bayes can handle it and is feasible, but no measurement invariance chi-square testing summary is provided

Slide 114 shows a practical complication when analyzing categorical factor indicators over time, namely having different number of response categories at different timepoints. For instance, some low or high categories may not have observations at the beginning or end of the time series.

The input on the previous slides showed an analysis for the first 7 timepoints. As shown in the first bullet, adding the 8th timepoint leads to an error in that this last timepoint doesn't have the same number of categories as the previous ones.

With ML and Bayes, this can be handled but not by WLSMV.

But ML requires numerical integration of high dimension and is therefore not possible.

Bayes can do it but does not produce invariance testing.

This may need further research.