4 EFA in a multiple-group setting

Outline

- EFA background
- EFA variations
  - ESEM, PSEM
  - Second-order, SEFA
  - Bi-factor, DSEFA
  - Target
- EFA in an SEM setting
  - MIMIC
  - EFA/CFA on EFA/CFA
- **EFA in a multiple-group setting**
  - EFA alignment
- EFA in a longitudinal setting
  - EFA longitudinal invariance testing
  - EFA longitudinal alignment
  - EFA growth modeling
- Further topics
- EFA theory

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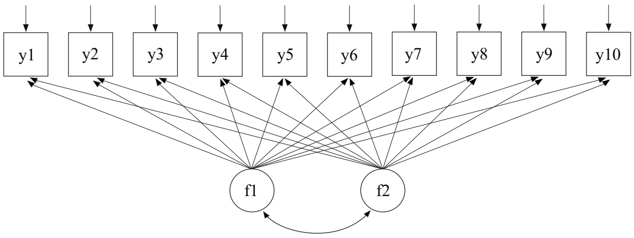
Slide 83 returns to the Outline of the presentation. After having covered EFA in an SEM setting, we are now turning to EFA in the common case of a multiple-group setting. This will also lead to discussing the useful technique of Alignment.

ESEM for Multiple-Group EFA: UG Ex 5.27 Part 3



TITLE:	this is an example of multiple-group EFA with continuous factor indicators with scalar measurement invariance
DATA:	FILE IS ex5.27.dat;
VARIABLE:	NAMES = y1-y10 group; GROUPING = group (1 = g1 2 = g2);
ANALYSIS:	MODEL = CONFIGURAL METRIC SCALAR;
MODEL:	f1-f2 BY y1-y10 (*1); ! scalar invariance

- Configural and metric invariance specifications shown in UG ex 5.27 part 1 and part 2

Slide 84 shows an Mplus User's Guide example where there are 2 EFA factors in 2 groups. The MODEL command specifies scalar invariance, that is, both loadings and indicator intercepts are held equal across the 2 groups. In the ANALYSIS command, however, we request testing of all 3 relevant measurement models by the specification model = configural metric scalar. How to specify configural and metric measurement models in the MODEL command is shown in parts 1 and 2 of the User's Guide example.

Measurement Invariance Testing Across Groups

MODEL FIT INFORMATION

Invariance Testing

Model	Number of Parameters	Chi-Square	Degrees of Freedom	P-Value
Configural	78	63.815	52	0.1261
Metric	62	77.687	68	0.1975
Scalar	54	82.326	76	0.2900

Models Compared	Chi-Square	Degrees of Freedom	P-Value
Metric against Configural	13.872	16	0.6083
Scalar against Configural	18.511	24	0.7775
Scalar against Metric	4.640	8	0.7953

Slide 85 shows the output for the invariance testing obtained by model = configural metric scalar. This is simulated data and all 3 models fit well. With real data, the comparison of models at the bottom part of the output is helpful for checking at which point misfit shows up. Metric invariance, that is, invariant loadings, may fit sufficiently well, but often scalar invariance does not fit well with its added requirement of invariant intercepts for the factor indicators.

4.1 Alignment

Alignment

- Metric and scalar measurement invariance often rejected
- The alignment model has the same fit as the configural model
- Alignment minimizes the amount of measurement noninvariance in intercepts and loadings by estimating group-varying factor means and variances
- The group-varying factor means and variances are not identified in the configural model - alignment avoids this problem by adding the necessary extra information via optimization of a simplicity criterion similar to EFA rotation criteria avoiding indeterminacies

Slide 86 turns to the topic of Alignment which is a very useful technique in multiple-group studies with both CFA and EFA measurement models.

The premise for Alignment is that metric and scalar measurement invariance is often rejected.

A solution to this is the fact that the alignment model only requires that a configural model fits well, that is, the pattern of zero loadings is invariant across groups.

Simply put, alignment minimizes the amount of measurement noninvariance in intercepts and loadings by estimating group-varying factor means and variances.

The interesting thing is that group-varying factor means and variances are not identified in the configural model. Alignment avoids this problem by adding the necessary extra information via optimization of a simplicity criterion similar to how EFA rotation criteria avoid the indeterminacies.

Alignment Papers

- Multiple-group alignment:
  - Asparouhov & Muthén (2014). Multiple-group factor analysis alignment. Structural Equation Modeling: A Multidisciplinary Journal, 21:4, 495-508.
  - Muthén & Asparouhov (2014). IRT studies of many groups: The alignment method. Frontiers in Psychology
  - Muthén & Asparouhov (2018). Recent methods for the study of measurement invariance with many groups: Alignment and random effects. Sociological Methods & Research, 47:4 637-664.
- Generalized multiple-group alignment - ASEM, AESEM (allowing cross-loadings, ESEM, factors regressed on factors, covariates):
  - Asparouhov & Muthén (2023). Multiple group alignment for exploratory and structural equation models. Structural Equation Modeling: A Multidisciplinary Journal, 30(2), 169-191.
  - Asparouhov & Muthén (2025). Alignment for multiple group second-order factor analysis. Mplus Web Notes: No. 27. Version 1. December 26, 2025.
- `https://www.statmodel.com/MeasurementInvariance.shtml`

Slide 87 gives a list of Alignment papers to study. They can all be found on the Special Mplus Topics page Measurement Invariance, which has many application papers.

Alignment Theory Briefly Stated: 2 Steps

- 1. Estimate the configural model:
  - Loadings ($\lambda_{configural}$) and intercepts ($\nu_{configural}$) free across groups, factor means fixed at zero in all groups, factor variances fixed at 1 in all groups
- 2. Do the alignment optimization:
  - Free the factor means and variances and choose their values to minimize the amount of noinvariance using a simplicity function
- In step 2, the factor means α_j and variances ψ_j are free parameters to be estimated, maintaining the configural model fit while obtaining the aligned λ_j and ν_j for group j:

$$\lambda_j = \lambda_{j,configural} / \sqrt{\psi_j} \quad (1)$$
$$\nu_j = \nu_{j,configural} - \alpha_j \, \lambda_{j,configural} / \sqrt{\psi_j} \quad (2)$$

It is not our intention to fully discuss Alignment in this presentation, but slide 88 gives the basic idea of the steps taken in this technique.

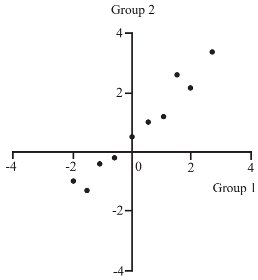
In Step 1, the configural model is estimated. Here, loadings and intercepts are free across groups, factor means are fixed at zero in all groups, and factor variances are fixed at 1 in all groups. This is the same specification as estimating each group separately.

Step 2 does the alignment optimization. Here, alignment frees the factor means and variances and choose their values to minimize the amount of noninvariance using a simplicity function.

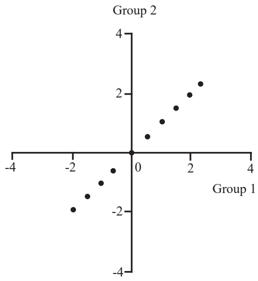
Equations 1 and 2 show the aligned loadings (λ) and indicator intercepts (ν) as functions of the configural counterparts from step 1 and the free factor variances and factor means from step 2.

Why “Alignment"? Intercept Invariance but Factor Diffs

- Intercepts of 10 indicators (dots)
- 1 factor
- 2 groups (axes)
- Factor means = 0, -1
- Factor variances = 1, 2



Unaligned: Configural model (factor mean=0, factor variance=1 in both groups)



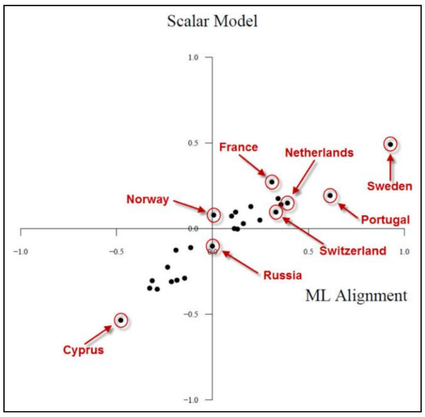
Aligned: Taking into account the group differences in factor means and variances

Slide 89 explains why the term alignment is used. The figures show 10 factor indicators measuring 1 factor in 2 groups. The intercepts of the indicators are the same in the 2 groups but the groups have different factor means and variances.

The top figure shows the first step of alignment where the factor means are set to zero in both groups and the factor variances are set to 1 in both groups.

The bottom figure shows the intercepts after alignment. Group differences in factor means and variances are allowed which makes the intercepts be perfectly aligned along the 45-degree line showing that they are the same in the 2 groups.

Alignment Application (Muthén & Asparouhov, 2018)



Factor means for 26 countries: Scalar versus Alignment

- Alignment agrees with Scalar that Sweden/Cyprus have the highest/lowest level of tradition nonconformity
- Alignment disagrees with Scalar regarding the difference between Portugal and Netherlands, between France and Switzerland, as well as between Norway and Russia

Slide 90 shows an application to a 26-country analysis where tradition nonconformity is measured. The figure shows the resulting factor means as obtained by alignment on the x-axis and by using a scalar model on the y-axis. As is often found, the two approaches do not give widely different results - the factor means largely fall on the 45-degree line. But, the two approaches do show some differences in the comparison of countries. For example, Alignment agrees with Scalar that Sweden/Cyprus have the highest/lowest level of tradition nonconformity. But Alignment disagrees with Scalar regarding the difference between Portugal and Netherlands in that these countries show a bigger difference as judged by the alignment axis than as judged by the scalar axis. Likewise, there is disagreement in the comparison between France and Switzerland, as well as between Norway and Russia.

Input for Alignment in the Single-Factor Case

```
DATA:          FILE IS ess05Traco.dat;
VARIABLE:      NAMES = country essround ipfrule ipmodst ipbhprp imptrad;
               USEVARIABLES = ipmodst imptrad ipfrule ipbhprp;
               MISSING = ipfrule-imptrad (7-9);
               GROUPING = country(2-18 21-28 30);

ANALYSIS:      ESTIMATOR = ML;
               ALIGNMENT = FIXED(22);

MODEL:         traco BY ipmodst-ipbhprp;

OUTPUT:        ALIGN; ! providing extra alignment output

PLOT:          TYPE = PLOT2;
```

- STANDARD ERROR COMPARISON INDICATES THAT THE FREE ALIGNMENT MODEL MAY BE POORLY IDENTIFIED. USING THE FIXED ALIGNMENT OPTION MAY RESOLVE THIS PROBLEM.
- Muthén & Asparouhov (2018): Tradition-conformative measures in 26 European countries

Slide 91 shows the input for the 26-country example discussed earlier. In the VARIABLE command, the GROUPING option specifies the countries involved.

ALIGNMENT is specified in the ANALYSIS command. Typically, the alignment uses the FREE option where no specific group is used as basis for the comparison. In this case, however, this gives the error message shown at the bottom:

STANDARD ERROR COMPARISON INDICATES THAT THE FREE ALIGNMENT MODEL MAY BE POORLY IDENTIFIED. USING THE FIXED ALIGNMENT OPTION MAY RESOLVE THIS PROBLEM.

With the FIXED option, a group with the smallest factor mean may be chosen as the basis for comparison. In this case, country 22 was chosen. Using FIXED without a qualifier uses the first group for comparison.

This example has only 1 factor which means that EFA is not relevant. The next example shows alignment with a 4-factor EFA.

AESEM: Alignment with ESEM

- Asparouhov & Muthén (2023): PISA example
  - 15-year old students, 30 OECD countries/groups, N = 249,840
  - Sampling weights, PSUs: Complex survey data (Asparohov, 2005)
  - 22 factor indicators, 4 factors, 3 covariates
- 4 Models:
  - No alignment, scalar invariance:
    - (1) SEM: CFA
    - (2) ESEM: EFA
      - More parameters than (1) due to cross-loadings
  - Alignment, configural invariance:
    - (3) ASEM: CFA
      - More parameters than (1) due to configural
    - (4) **AESEM**: EFA
      - More parameters than (1) due to cross-loadings and configural

Slide 92 shows a large example from the PISA study (Programme for International Student Assessment) which is an international study that evaluates the knowledge and skills of 15-year-old students in reading, mathematics, and science. The sample consists of about 250 thousand students from 30 countries. The data features sampling weights, PSUs (primary sampling units) and school clusters, that is, complex survey data features. The implementation of such features is discussed in Asparouhov (2005).

The dataset considered here has 22 indicators of 4 factors. In addition, 2 covariates are added to the alignment analysis. Four models are compared in the Asparouhov-Muthén (2023) paper:

No alignment, scalar invariance:
(1) SEM: CFA
(2) ESEM: EFA
- We note that (2) has more parameters than (1) due to cross-loadings

Alignment, configural invariance:
(3) ASEM: CFA
- We note that (3) has more parameters than (1) due to configural
(4) AESEM: EFA
- We note that (4) has more parameters than (1) due to cross-loadings and configural

The question is which of the 4 models is best for the PISA data.

Comparison of Multiple-Group Analyses for PISA Example

- Table 9 in Asparouhov & Muthén (2023): AESEM fits best

Model	SEM	ESEM	ASEM	**AESEM**
Number of parameters	1,476	1,530	2,520	4,140
Chi-square	162,449	145,919	85,308	62,943
Degrees of freedom	8,754	8,700	7,710	6,090
BIC	10,404,759	10,376,631	10,286,767	**10,268,659**
CFI	0.92	0.93	0.96	0.97
TLI	0.92	0.93	0.95	0.96
SRMR	0.049	0.045	0.029	0.019
RMSEA	0.046	0.044	0.035	0.034

- SEM: scalar invariance, CFA
- ESEM: scalar invariance, EFA.
- ASEM: alignment (configural), CFA.
- AESEM: alignment (configural), EFA

Slide 93 compares the 4 approaches shown as different columns in the table. You see the increase in parameters going from left to right with AESEM having over 4 thousand parameters. Chi-square is difficult to work with here given the large sample size of 250 thousand students. CFI improves going from left to right as expected with increasing number of parameters. BIC is useful here because it has a penalty for the number parameters. BIC points to AESEM as the best model with the lowest BIC. AESEM uses configural invariance for an EFA measurement model.

The AESEM input is shown on the next slide.

Input for AESEM Analysis of the PISA Example Asparouhov & Muthén (2023)

```
DATA:       FILE = pisa06_alignment_final_data_r.dat;

VARIABLE:   NAMES = schoolid stidstd country oecd w_fstuwt st16q01-st16q05
            st17q01-st17q08 st18q01-st18q10 st19q01-st19q06 st21q01-st21q08
            st29q01-st29q04 st35q01-st35q05 st37q01-st37q06 pv1scie gender ses
            cntgen zpvliscie zgenders zses;
            USEVARIABLES = st16q01-st16q05 st35q01-st35q05 st29q01-st29q04
            st17q01-st17q08 gender zses zpvliscie;
            WEIGHT = w_fstuwt;
            CLUSTER = schoolid;
            MISSING = .;
            GROUPING = country(30);

DEFINE:     schoolid=(country*10000)+schoolid; ! creates unique schoolid

ANALYSIS:   TYPE = COMPLEX;
            ALIGNMENT = FIXED; ! group 1 chosen for comparison
            STARTS = 30; ASTARTS = 100; ! starts for step 1 and step 2 (alignment)

MODEL:      f1-f4 BY st16q01-st17q08 (*1);
            f1-f4 ON gender zses zpvliscie;
```


Slide 94 shows the input for the AESEM analysis just discussed. It specifies that there are 30 countries. TYPE = COMPLEX handles the sampling weights and the clusters. The fixed alignment option is chosen here which implies that the factor means/intercepts are fixed at zero in the first group. Given the complexity of the analysis, 30 sets of random starting values are used together with 100 alignment starts.

The modeling uses an ESEM setup for the 4 factors behind the 22 indicators. The 4 factors are regressed on 3 covariates.

This is a time-consuming analysis given the large number of parameters.

The results are discussed in Section 4 of Asparouhov & Muthén (2023).