

### 3 EFA in an SEM setting

#### Outline

- EFA background
- EFA variations
  - ESEM, PSEM
  - Second-order, SEFA
  - Bi-factor, DSEFA
  - Target
- **EFA in an SEM setting**
  - MIMIC
  - EFA/CFA on EFA/CFA
- EFA in a multiple-group setting
  - EFA alignment
- EFA in a longitudinal setting
  - EFA longitudinal invariance testing
  - EFA longitudinal alignment
  - EFA growth modeling
- Further topics
- EFA theory

Slide 66 returns to the overall Outline of the presentation, having covered EFA Variations. We now go beyond the focus on EFA of a measurement instrument in isolation and turn to the topic of EFA in a structural equation modeling setting. We will cover the case of MIMIC modeling, that is, adding covariates as well as discussing EFA in general SEM settings.

## ESEM and PSEM Sources

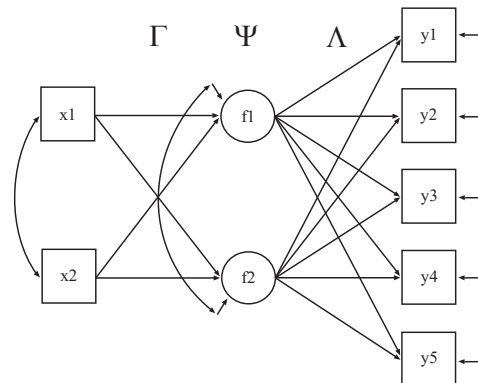
- ESEM:
  - Asparouhov & Muthén (2009). Exploratory structural equation modeling. *Structural Equation Modeling: A Multidisciplinary Journal*
  - Application papers at:  
<http://www.statmodel.com/ESEM.shtml>
- PSEM:
  - Asparouhov & Muthén (2024, 2025, 2026)
  - Muthén (2005)
  - Special Mplus Topics:  
<https://www.statmodel.com/psem.shtml>

Slide 67 lists some references for what we will talk about. ESEM will be a key approach for EFA in an SEM setting and this idea was presented in the 2009 Asparouhov-Muthén paper. As you see here, ESEM has its own Special Mplus Topics web page where you find several application papers.

The PSEM approach is also essential for EFA in an SEM setting. PSEM developments have appeared in papers and talks which are all listed on our Special Mplus Topics page for PSEM. This includes PSEM theory in our papers from 2024, 2025, and 2026 listed in the references.

### 3.1 ESEM MIMIC

#### Example: ESEM MIMIC



- Step 1: Unrotated loadings model, estimating  $\Lambda$  with  $\Psi = I$  (standardized, uncorrelated factors), and the  $\Gamma$  regressions of factors on covariates
- Step 2: EFA rotation of the y measurement part gives new  $\Lambda$ ,  $\Psi$ , and a corresponding transformation is applied to the estimated  $\Gamma$  from Step 1

Slide 68 shows an example with an EFA measurement model that has 2 factors and 6 indicators, to which we add 2 covariates  $x_1$  and  $x_2$ . The loading matrix is called  $\Lambda$ , the factor covariance matrix is called  $\Psi$  and the regression matrix is called  $\Gamma$ .  $\Psi$  is now actually a residual covariance matrix for the factors. This is called a MIMIC model where MIMIC stands for multiple indicators, multiple causes.

Typically, MIMIC modeling is done by using a CFA measurement model but here the ESEM approach makes it possible to use an EFA measurement model. ESEM uses two steps:

In Step 1, an unrotated loadings model estimates  $\Lambda$  with  $\Psi = I$  (so standardized, uncorrelated factors), and the  $\Gamma$  regressions of factors on covariates.

In Step 2, an EFA rotation is made of the y measurement part and gives new  $\Lambda$  and  $\Psi$ , after which a transformation is applied to the estimated  $\Gamma$  from Step 1.

## ESEM MIMIC Inputs

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### ESEM MIMIC

MODEL:

```
f1-f5 BY y1-y15 (*1);  
f1-f5 ON age;
```

---

### ESEM MIMIC with rescovs

MODEL

```
f1-f5 BY y1-y15 (*1);  
y1 y5 y9 y12 WITH y1 y5 y9 y12;  
f1-f5 ON age;
```

---

### Bi-Factor ESEM MIMIC

ANALYSIS:

```
ROTATION = BI-GEOMIN;
```

MODEL:

```
fg f1-f4 BY y1-y15(*1);  
fg-f4 ON age;
```

---

Slide 69 shows the generality of the ESEM MIMIC approach in three variations of the input: the EFA MIMIC just discussed, a version that adds some residual covariances among the indicators, and MIMIC with a bifactor measurement model where the general factor is also regressed on the covariates.

## ESEM MIMIC Examples

- User's Guide ex 5.24: ESEM
  - f1-f2 BY y1-y8 (\*1); f1-f2 ON x1-x2;
- Asparouhov-Muthén (2026): PSEM SEFA for HS19 with second-order factor regressed on grade and gender
- Muthén (2025): ESEM and PSEM with WLSMV for binary factor indicators of antisocial behavior
  - ESEM MIMIC analysis: EFA measurement model with factors regressed on covariates and covariates influencing indicators directly

Slide 70 lists some applications of ESEM MIMIC.

Apart from the User's Guide example 5.24, our 2026 paper illustrates the use of a SEFA measurement model in a PSEM analysis where the second-order factor is regressed on covariates.

The Muthén 2025 handbook chapter shows ESEM together with PSEM with WLSMV estimation for binary indicators. Both the factors and the indicators are regressed on covariates. The indicator regressions capture measurement non-invariance. This example will be shown next.

## Input for ESEM MIMIC with 17 Items, 3 Factors, and 11 Covariates: ASB Example (Muthén, 2025)

```
VARIABLE:      CATEGORICAL = property-gambling;

ANALYSIS:      ESTIMATOR = WLSMV;

MODEL:         ! 3-factor EFA measurement model:
               f1-f3 BY property-gambling (*1);

               ! EFA factors regressed on covariates:
               f1-f3 ON sex black hisp single divorce dropout
               college onset age94 dep abuse;

OUTPUT:        TECH1 TECH4 TECH10
               STANDARDIZED RESIDUAL;

PLOT:          TYPE = PLOT3;
```

Slide 71 shows the input used in the 2025 paper. The example refers to antisocial behavior where 17 binary variables are used as indicators of 3 factors regressed on 11 covariates.

The VARIABLE command specifies the indicators as categorical and the ANALYSIS command requests weighted least squares estimation. The EFA specification uses the ESEM approach, followed by the regression of the EFA factors on the covariates and using the 2 rotation steps outlined earlier on slide 68.

- ESEM-PSEM analysis: Direct effects added from covariates to factor indicators to capture measurement non-invariance
  - Using PSEM ALF priors makes it possible to identify all direct effects
  - The ALF prior variance is chosen so that the loglikelihood is the same as the "null" model which in this case is ESEM with regressions of all indicators on all covariates but no regressions of the factors on the covariates
- 17 factor indicators and 11 covariates results in 187 possible direct effects, of which PSEM finds that 40 are significant
  - Freeing these 40 direct effects changes the relations between the factors and the covariates

Slide 72 discusses the addition of direct effects from the covariates to the indicators. The presence of such effects reflect measurement non invariance.

The direct effects are not identified when added all at the same time, but PSEM using ALF makes it possible to identify them.

Here, it is important to choose the prior variance of ALF properly. It should be chosen such that the loglikelihood is the same as the so called null model which in this case is the model with all direct effects but no effects on the factors.

There is a total of 187 possible direct effects. The PSEM ALF approach avoids a long stepwise search for significant effects and includes all of them in one single run. In this case, 40 effects were significant. These effects can then be freed in a new run to check to which extent the relations have changed between the factors and the covariates. The paper, which is on the Mplus website, gives further details and discusses the interpretation of the findings.

The input is shown on the next slide.

## Input for PSEM Direct Effects for ASB

```
USEVARIABLES = property-gambling sex black hisp single
divorce dropout college onset age94 dep abuse;
CATEGORICAL = property-gambling;

ANALYSIS: ESTIMATOR = WLSMV;

MODEL: f1-f3 BY property-gambling (*1);
f1-f3 ON sex black hisp
single divorce dropout
college onset age94 dep abuse;
property-gambling ON sex-abuse (p1-p187);

MODEL PRIORS: p1-p187 ALF(0,1);

OUTPUT: TECH1 TECH4 TECH10
STANDARDIZED RESIDUAL;

PLOT: TYPE = PLOT3;
```

Slide 73 gives the input for the analysis just discussed. The bolded lines show the changes made to the regular ESEM MIMIC. The direct effects are labeled and then given ALF priors with mean zero and variance 1.

## 3.2 EFA/CFA on EFA/CFA

### Outline

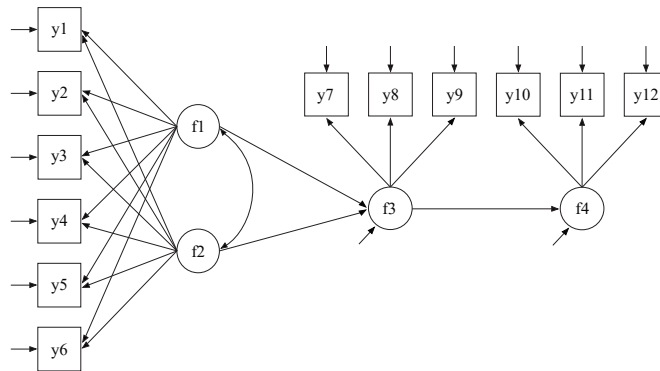
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Slide 74 returns to the outline of the presentation. While MIMIC is a structural equation model, we are now turning to more general SEM settings with various combinations of EFA and CFA measurement models.

- CFA on EFA
  - CFA on EFA and X and Y
  - EFA on CFA
  - EFA on CFA and X and Y
  - EFA on EFA
- 
- Factors of an EFA must be in a block by themselves, but you can have several EFA blocks

Slide 75 lists some variations of combinations of EFA and CFA measurement models in structural equation models that can be handled by Mplus. For example, there can be a CFA model for the dependent variable side of an SEM with EFA factors on the independent variable side. The reverse is also possible. Other independent or dependent variables can be added - here marked as X and Y. We will also see examples of EFA factors regressed on EFA factors, either in two different EFA blocks or within the same block. Some examples will clarify this.

## CFA Regressed On EFA: UG Ex5.25



TITLE: this is an example of SEM with EFA and CFA factors with continuous factor indicators

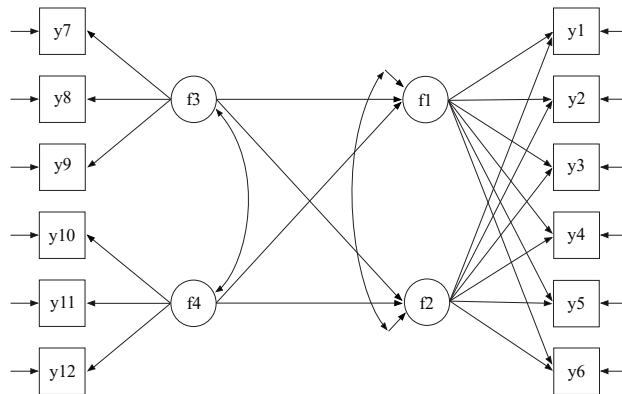
DATA: FILE IS ex5.25.dat;

VARIABLE: NAMES = y1-y12;

MODEL: f1-f2 BY y1-y6 (\*1); ! EFA factors  
f3 BY y7-y9; ! CFA factor  
f4 BY y10-y12; ! CFA factor  
f3 ON f1-f2;  
f4 ON f3;

Slide 76 shows an example of CFA factors regressed on EFA factors. This is the Mplus User's Guide example 5.25. The MODEL command shows that the f1, f2 factors are part of an EFA block, while f3 and f4 are CFA factors.

## EFA Regressed On CFA: UG Ex5.25 Reversed

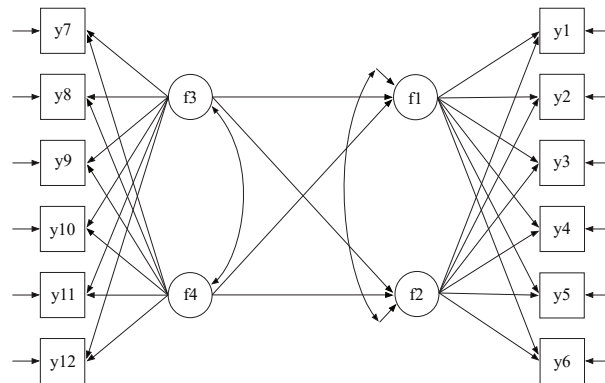


MODEL:

f1-f2 BY y1-y6 (\*1); ! EFA factors  
f3 BY y7-y9; ! CFA factor  
f4 BY y10-y12; !CFA factor  
f1-f2 ON f3 f4;

Slide 77 shows the reverse situation with an example of EFA factors regressed on CFA factors. The EFA factors f1 and f2 are now on the dependent variable side of the model.

## EFA Regressed On EFA: UG Ex5.25 Modified: Two EFA Blocks

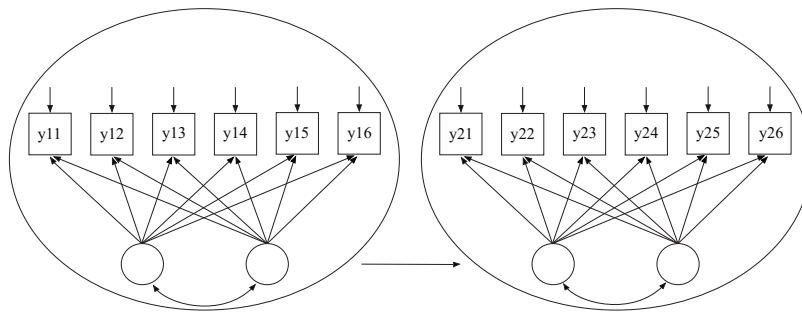


MODEL:

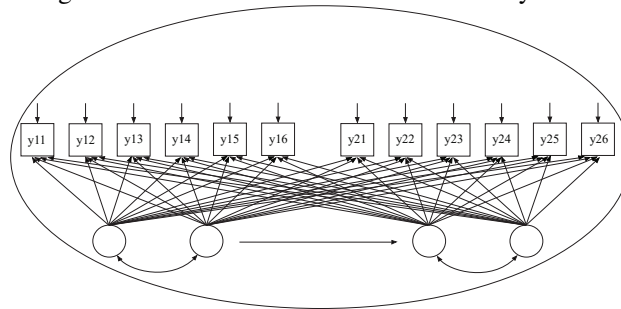
f1-f2 BY y1-y6 (\*1);  
 f3-f4 BY y7-y12 (\*2);  
 f1-f2 ON f3 f4;

Slide 78 shows an example of EFA factors regressed on EFA factors. This is an example of using two EFA blocks. Here, f1 and f2 are in EFA block 1, designated by (\*1), while f3 and f4 are in EFA block 2 designated by (\*2).

## ESEM vs PSEM



Regression between 2 blocks: Can be done by ESEM



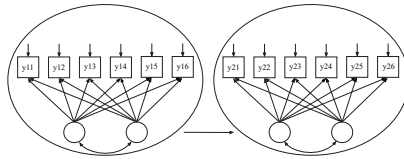
Regression within 1 block: Can only be done by PSEM

Slide 79 shows the difference between using ESEM and using PSEM. The top part of the slide shows regression between two different EFA blocks just as in the example of the previous slide. This can be done by ESEM.

The bottom part of the slide shows that all four factors influence all 12 indicators. This represents a single EFA block with a regression between the EFA factors. Such a within-block regression can only be carried out with the help of PSEM.

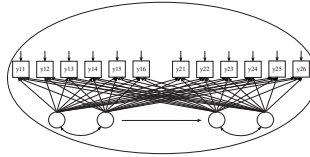
The corresponding two inputs are shown on the next slide.

## ESEM vs PSEM Input



Regression between 2 blocks: Can be done by ESEM

f1-f2 BY y11-y16 (\*1);  
 f3-f4 BY y21-y26 (\*2);  
 f3-f4 ON f1-f2;



Regression within 1 block: Can only be done by PSEM

f1-f4 BY y11-y26\* (p1-p48); f1-f4@1;  
 f3-f4 ON f1-f2;

MODEL PRIORS:

p1-p48~GEOMIN(4,1,0.0001);

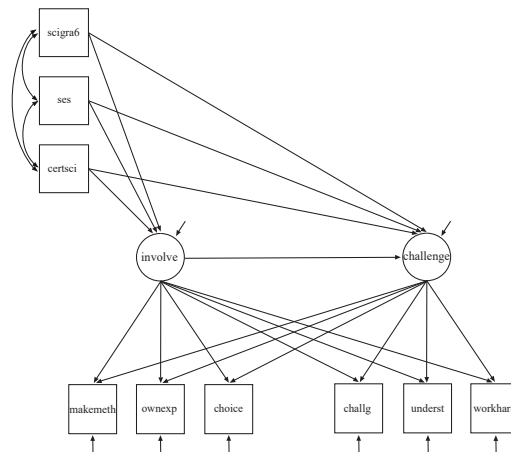
Slide 80 shows the input for the two situations just discussed.

For the top case, the input shows two EFA blocks expressed in ESEM terms.

For the bottom case, a single EFA block is specified using the PSEM GEOMIN approach to EFA - which was one of the 3 EFA approaches we listed on slide 8 and used on slide 14.

The MODEL PRIORS command specifies GEOMIN rotation for the set of 4 factors. The prior variance is set at 1 and a small epsilon value is chosen. The GEOMIN prior settings are shown in the EFA Theory section of the presentation.

## EFA in SEM Using PSEM: Science Example



MODEL:

```

involv BY makemeth*1 ownexp*1 choice*1 challg*0 underst*0 workhard*0 (p1-p6);
chall BY makemeth*0 ownexp*0 choice*0 challg*1 underst*1 workhard*1 (p7-p12);
involv@1 chall@1; ! 1 block: the 2 factors measured by makemeth-workhard
chall ON involv; involv ON scigra6-certsci; chall ON scigra6-certsci*0;
  
```

ANALYSIS:

```
! ITERATIONS = 10000; STARTS = 50; CONVERGENCE = 0.000001;
```

MODEL PRIORS:

```
p1-p12~GEOMIN(2,1,0.0001);
```

Slide 81 shows an example of regression within an EFA block. The example is based on the Science data used in Muthén & Asparouhov (2012). A subset of this was used as a PSEM illustration in the Asparouhov (2023) M3 workshop which you find on the next slide.

The two EFA factors challenge and involve are measured by six indicators. The challenge factor is regressed on the involve factor, that is, the model has a within-block regression like at the bottom of slide 80. Both factors are regressed on three covariates.

The input gives labels to the 12 loadings which are given GEOMIN priors. The number 2 refers to the number of factors, 1 is the prior variance, and epsilon is set to a small number which is required in this situation.

## PSEM Presentations and Web Site

- Asparouhov & Muthén (2024). Penalized structural equation models. *Structural Equation Modeling*. 31, 429-454.
- Asparouhov & Muthén (2025). Methodological advances with penalized structural equation models. *Structural Equation Modeling*. 32, 688-716.
- Asparouhov (2023) M3 conference workshop: <https://www.statmodel.com/download/M3TeachingSlides.pdf>
- Asparouhov (2023) M3 conference talk: <https://www.statmodel.com/download/M3Talk.pdf>
- All of the above found at the web site **Special Mplus Topics, PSEM**: <https://www.statmodel.com/psem.shtml>

Slide 82 gives a list of references for further PSEM readings. As you see at the bottom, there is a special web site for PSEM.