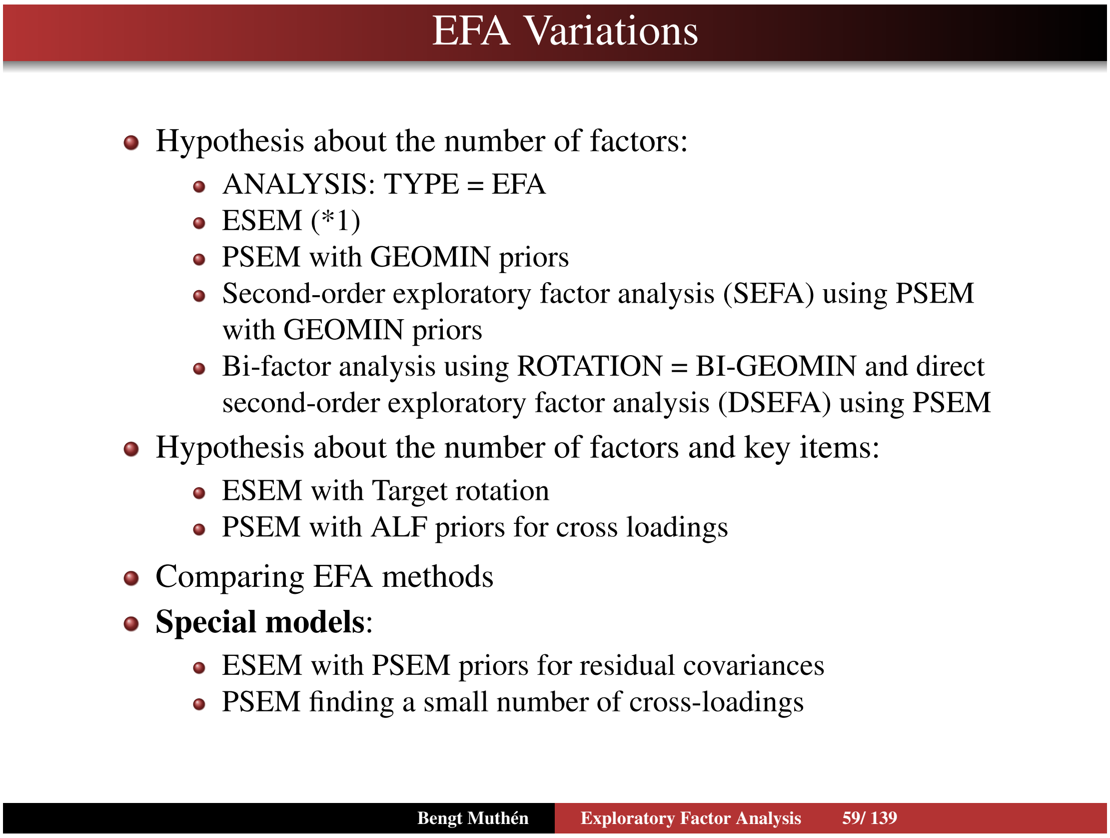
2.4 Special models



Slide 59 again shows the overview of EFA Variations. We now turn to the last 2 items on the list under Special models: ESEM where all residual covariances are included and PSEM searching for a small number of cross loadings.

ESEM with PSEM Priors for Residual Covariances

- PSEM makes it possible to allow **all** residual covariances
- Significant residual covariances can then be freed

```
MODEL:          f1-f2 BY y1-y10 (*1);
                y1-y10 WITH y1-y10 (c1-c45);

MODEL PRIORS:   c1-c45~ALF(0,1);
```

- See Section 4.5 of Asparouhov & Muthén (2024)

Slide 60 shows that it is possible to include residual covariances in an EFA by using PSEM. In the previous British Household Panel example we used ESEM to include some residual covariances. With PSEM, however, we can include all of them. This makes it possible to see which of them are significant and should be freed.

As the input shows, ALF priors can be applied to all the residual covariances. These priors push the residual covariances toward the ALF prior mean of zero. If the dataset clearly requires a residual covariance, it will overwhelm the prior and show a substantial non-zero residual covariance estimate.

The technical background for this is discussed in section 4.5 of the 2024 Asparouhov-Muthén article on PSEM.

It is not recommended to use this approach in conjunction with more complex models such as SEFA and DSEFA. The alternative approach of using modindices with regular EFA was presented on slide 42.

PSEM Used for Finding the Smallest Number of Essential Cross-Loadings: Transition from EFA to CFA

- Strategy:
  - Start from an EFA and check the number of significant cross loadings
  - Continue with a PSEM CFA that adds cross-loadings with ALF priors
    - Specify ALF priors with a variance that makes the loglikelihood match that of EFA (typically variance = 1.0)
    - Specify ALF priors for cross-loadings with decreasing variance until the loglikelihood decreases significantly
  - Do a CFA which frees the cross-loadings that are significant in the previous run and see if BIC has improved compared to EFA

Slide 61 shows a different use of PSEM. It can answer the question of which is the smallest set of essential cross loadings when transitioning from an EFA to a CFA. BIC is used as the basis for the decision.

The strategy is to start from an EFA where you check the number of significant cross loadings.

You then continue with a PSEM CFA that adds cross-loadings with ALF priors.

- First you specify ALF priors with a variance that makes the loglikelihood match that of EFA (typically variance = 1.0).

- Then you specify these ALF priors for cross-loadings with decreasing variance until the loglikelihood decreases substantially.

Finally, you do a CFA where you free the cross-loadings that are significant in the previous run and see if BIC has improved compared to EFA.

Example: NELS Data (N = 5198)

- National Education Longitudinal Study, eighth graders in urban Catholic and Public schools (Muthén et al., 1997)

Variable	Reading	Math	Science	HCG
Y1	X	0	0	0
Y2	X	0	0	0
Y3	X	0	0	0
Y4	X	0	0	0
Y5	X	0	0	0
Y6	0	X	0	0
Y7	0	X	0	0
Y8	0	X	0	0
Y9	0	X	0	0
Y10	0	0	X	0
Y11	0	0	X	0
Y12	0	0	X	0
Y13	0	0	X	0
Y14	0	0	0	X
Y15	0	0	0	X
Y16	0	0	0	X

- Reading: literature, science, poetry, biography, history. Math: algebra, artihmetic, geometry, probability. Science: earth, chemistry, life, methods. HCG: history, geography, citizenship

Slide 62 shows the expected loading pattern for a measurement instrument used in the National Education Longitudinal Study of eighth graders in the US. The sample size is 5,198. Four factors are expected. The 16 factor indicators are described at the bottom of the table.

PSEM Input for NELS

- Specify the CFA factors
- Label the cross loadings
- Give ALF priors to the cross loadings

```
MODEL:
        f1 BY y1-y5*1
          y6-y16*0 (a6-a16);
        f2 BY y6-y9*1
          y1-y5*0 (b1-b5)
          y10-y16*0 (c10-c16);
        f3 BY y10-y13*1
          y1-y9*0 (d1-d9)
          y14-y16*0 (e14-e16);
        f4 BY y14-y16*1
          y1-y13*0 (f1-f13);
        f1-f4@1;
MODEL
PRIORS:
        a6-f13~ALF(0,1);
```


Slide 63 shows the input for the PSEM part of the exploration of essential cross loadings.
You specify the CFA factor pattern, label the cross loadings, and give ALF priors to those cross loadings.

NELS 4-Factor Solutions using EFA and PSEM

Model	# par's	LL	BIC	χ^2	Df	X-loads
1. EFA	90	-124527	249823	157	62	13
2. CFA	54	-124651	249764	394	98	0
3. CFA + x-loads PSEM ALF v=1.0	90	-124527	249823	156	62	11
4. CFA + x-loads PSEM ALF v=0.1	90	-124538	249847	187	62	6
5. CFA + 13 free x-loads of M1	67	-124545	249664	189	85	13
6. CFA + 11 free x-loads of M3	65	-124556	249668	211	87	11
7. CFA + 6 free x-loads of M4	60	-124566	**249645**	229	92	6

Slide 64 shows a series of models using EFA, CFA, and CFA with PSEM. Of particular importance are the loglikelihood (LL) and BIC values.

Model 1 is a 4-factor EFA which has 13 significant cross loadings.

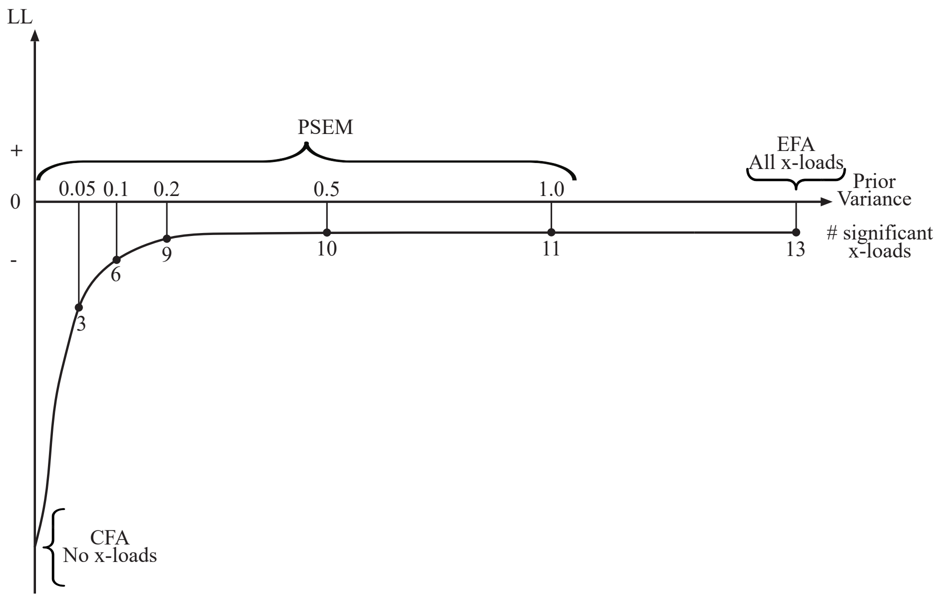
Model 2 is a regular CFA. The CFA model has better (lower) BIC than the EFA.

Model 3 is a CFA adding all cross-loadings using ALF priors. The variance 1 results in the same model fit as the Model 1 EFA as evidenced by the log likelihood (LL). This results in 11 significant cross loadings, representing a modest reduction from the 13 found with EFA. Having found this model fit equivalence with EFA, smaller variances for the ALF priors can be explored.

Model 4 uses ALF prior variance of 0.1. As expected, this worsens the log likelihood. To gauge the magnitude of the drop, it is useful to compare it to the distance between the EFA and CFA log likelihoods. In those terms, the drop is only 8.9%. For this model, the number of significant cross loadings is only 6. Models with small log likelihood drops and a number of significant cross loadings smaller than that of EFA provide a basis for a follow-up CFA that may have a competitive BIC as illustrated in the bottom part of the table.

The bottom part of the table adds significant cross loadings to the regular CFA Model 2 based on models 1, 3, and 4. These 3 models give better (lower) BIC values than any of the models in the top part. Of particular importance is that Model 7 with only 6 cross loadings gives the best BIC and is therefore the model of choice.

LL Curve for NELS



Slide 65 shows a log likelihood curve for the different analyses on the previous slide. At top right is the EFA model which has all cross loadings of which 13 are significant. It can be seen as a PSEM model where the ALF prior variances are very large so that the priors essentially have no effect.
Bottom left is the CFA model which has no cross loadings. In between, there is a series of PSEM models with varying ALF prior variance and corresponding number of significant cross loadings. The plot shows the chosen model which has prior variance 0.1 with 6 significant cross loadings, representing the 8.9% drop in the log likelihood, and receiving the best BIC.