Wheaton et al. 46 Years Later: A Better Fitting Longitudinal SEM

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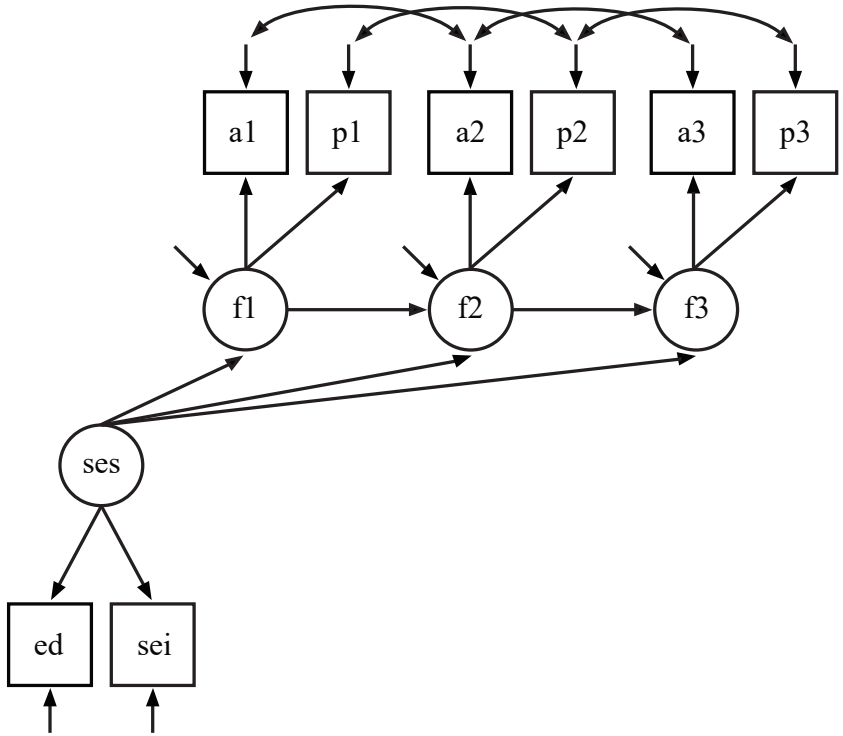
Outline

- History of the Wheaton et al. (1977) model
- Modeling ideas from multilevel factor analysis
- A new longitudinal SEM for the Wheaton et al. data
- Mplus scripts

History: 46 Years Ago at the University of Wisconsin - Madison, Wisconsin 1974-75

Wheaton, B., Muthén, B., Alwin, D., & Summers, G. (1977). Assessing reliability and stability in panel models. In D. R. Heise (Ed.), Sociological Methodology 1977 (pp. 84 - 136). San Francisco: Jossey-Bass, Inc.

"Wheaton et al." 1977 Structural Equation Model of the Stability of Alienation 1996-1971



- Anomia and Powerlessness as indicators of alienation
- Interest in stability estimates 1966 to 1967 and 1967 to 1971 while taking measurement error into account

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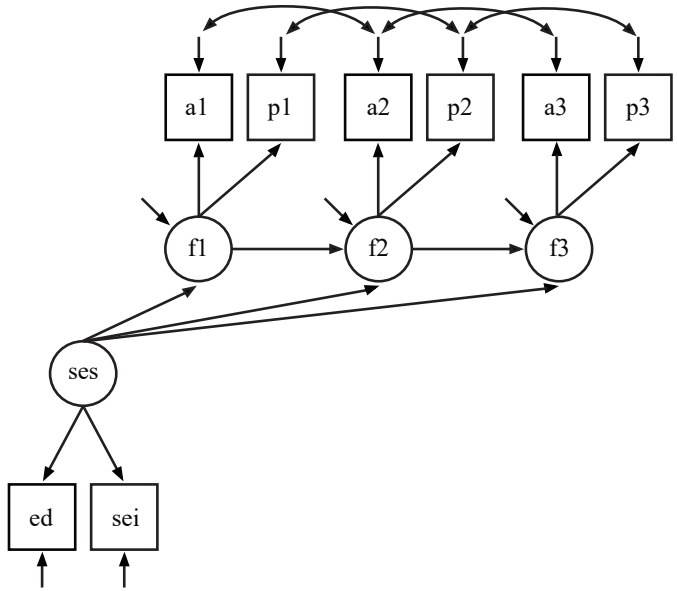

- Critique of the model (see also Short Course Topic 1 video and slides 231-238):
  - Only 2 factor indicators: Model mis-specification risk
  - Reflective SES factor: Formative might be better

Model Fit Results (N = 932)

- Model version using a standard specification:
  - Metric invariance for the measurement part (time-invariant factor loadings)
  - Correlated residuals for each factor indicator and adjacent time points

Model	# par's	logL	BIC	Chi-2	df	p	RMSEA	CFI
Classic	31	-19455	39121	90.04	13	0.00	0.080	0.979

Model Fit Results: Classic vs New (N = 932)

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Multilevel Factor Analysis Origins

$$
\begin{aligned}
Y_{ij} &= \nu + Y_{B_j} + Y_{W_{ij}} \\
&= \nu + \underbrace{\Lambda_B f_{B_j} + \varepsilon_{B_j}} + \underbrace{\Lambda_W f_{W_{ij}} + \varepsilon_{W_{ij}}},
\end{aligned}
$$

with covariance structure $V(Y_{ij}) = \Sigma_B + \Sigma_W$ where each covariance matrix has a factor model structure, $\Lambda\,\Psi\,\Lambda' + \Theta$.

- Cronbach (1976). Research on classrooms and schools: Formulation of questions, design, and analysis. Stanford University, School of Ed
- Härnqvist (1978). Primary mental abilities of collective and individual levels. Journal of Educational Psychology
- Goldstein & McDonald (1988). A general model for the analysis of multilevel data. Psychometrika
- McDonald & Goldstein (1989). Balanced versus unbalanced designs for linear structural relations in two-level data. British Journal of Mathematical and Statistical Psychology

Multilevel Factor Analysis Continued

- Muthén (1989). Latent variable modeling in heterogeneous populations. Psychometrika
- Muthén (1991). Multilevel factor analysis of class and student achievement components. Journal of Educational Measurement
- Muthén (1994). Multilevel covariance structure analysis. In J. Hox & I. Kreft (eds.), Multilevel Modeling, a special issue of Sociological Methods & Research
- Students within classrooms or schools. Achievement testing
  - There are typically fewer between-level factors fb than within-level factors fw
  - fb interpretation different from fw interpretation, e.g.:
    - fw: Math and reading skill factors
    - fb: School excellence

Random Intercept View of Two-Level Factor Analysis

The two-level factor analysis model (one factor on each level),

$$y_{ij} = \nu + \lambda_B f_{B_j} + \varepsilon_{B_j} + \lambda_W f_{W_{ij}} + \varepsilon_{W_{ij}}$$

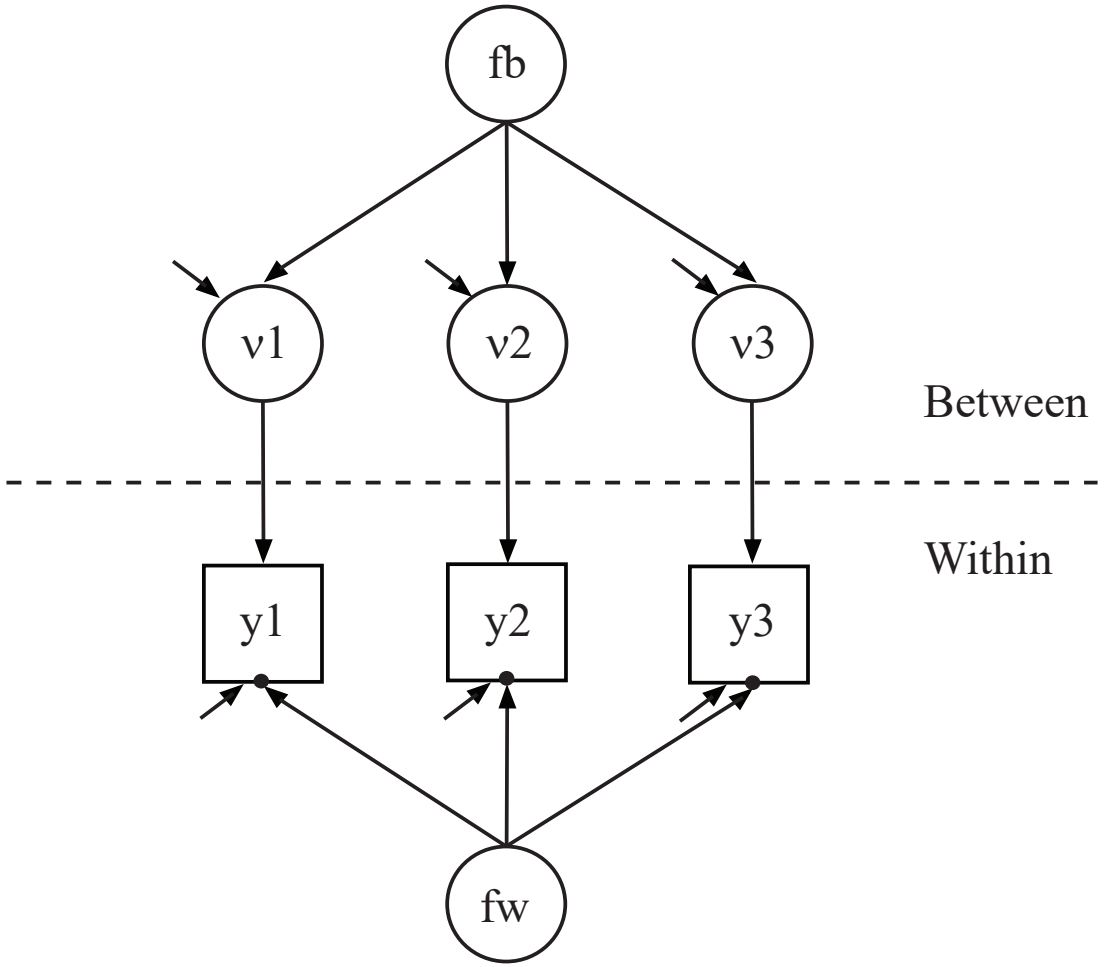
expressed in terms of level 1 and level 2 (within and between),

$$Level\ 1: \ y_{ij} = \nu_j + \lambda_W f_{W_{ij}} + \varepsilon_{W_{ij}}, \quad (1)$$

$$Level\ 2: \ \nu_j = \nu + \lambda_B f_{B_j} + \varepsilon_{B_j}. \quad (2)$$

- Two key aspects:
  - Two-level factor analysis can be viewed as a random intercept model where the intercepts have a factor structure
  - Because the random intercepts appear for the factor indicators y, measurement non-invariance is allowed

Multilevel Factor Analysis: Model Diagram



$$Level\ 2: \quad \nu_j = \nu + \lambda_B f_{B_j} + \varepsilon_{B_j}$$

$$\Sigma_B$$

$$Level\ 1: \quad y_{ij} = \nu_j + \lambda_W f_{W_{ij}} + \varepsilon_{W_{ij}}$$

$$\Sigma_W$$

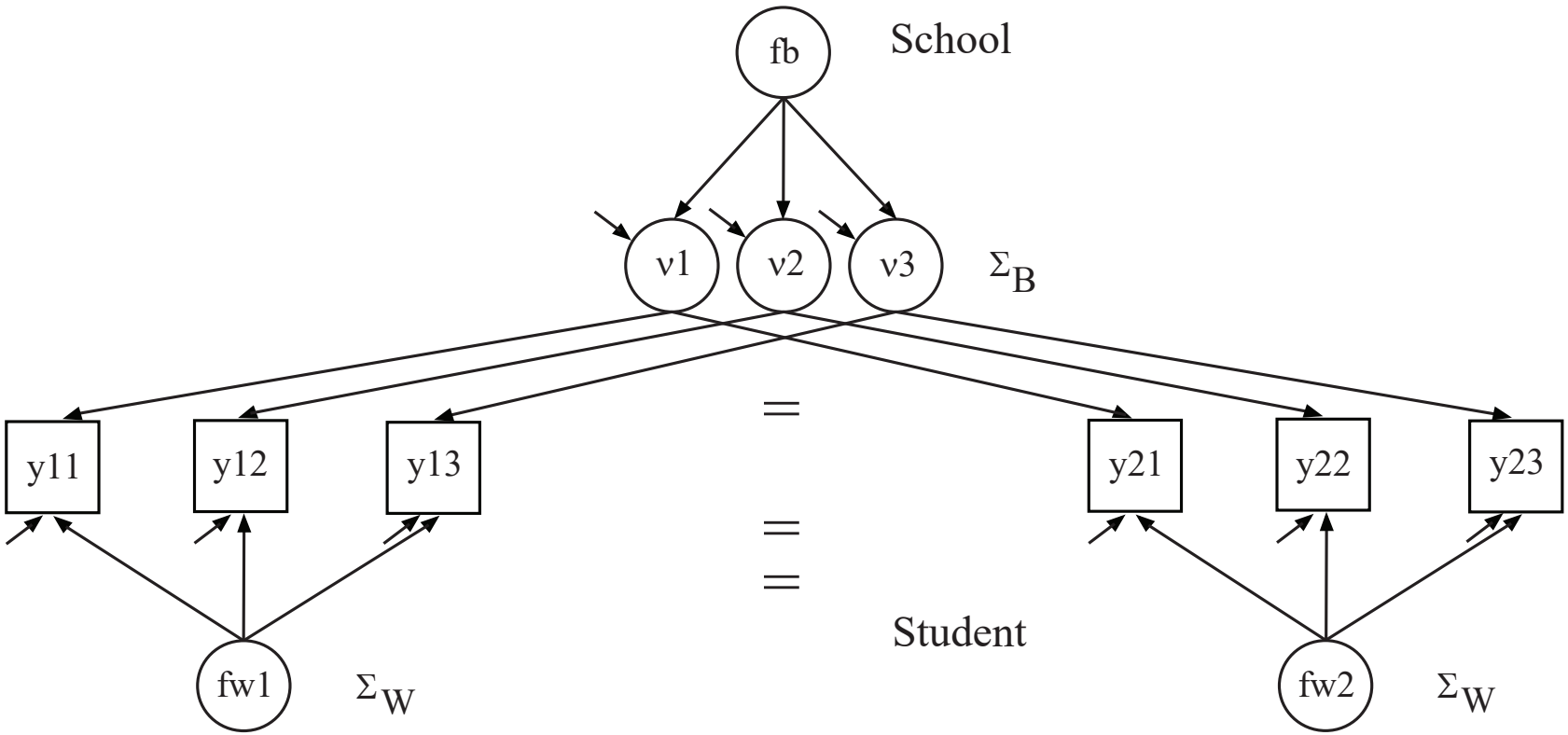
Going Deeper into Multilevel Factor Analysis: Covariance Structure for Students Within Schools Displaying the Data for Each Student

$$y_{ij} = \nu + \lambda_B f_{B_j} + \varepsilon_{B_j} + \lambda_W f_{W_{ij}} + \varepsilon_{W_{ij}}$$

School	Student	Data	Covariance Structure				
1	1	$\mathbf{y_{11}}$	$\mathbf{\Sigma_w + \Sigma_B}$	symmetric			
1	2	$\mathbf{y_{12}}$	$\mathbf{\Sigma_B}$	$\mathbf{\Sigma_w + \Sigma_B}$			
1	3	$\mathbf{y_{13}}$	$\mathbf{\Sigma_B}$	$\mathbf{\Sigma_B}$	$\mathbf{\Sigma_w + \Sigma_B}$		
2	1	$\mathbf{y_{21}}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{\Sigma_w + \Sigma_B}$	
2	2	$\mathbf{y_{22}}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{\Sigma_B}$	$\mathbf{\Sigma_w + \Sigma_B}$

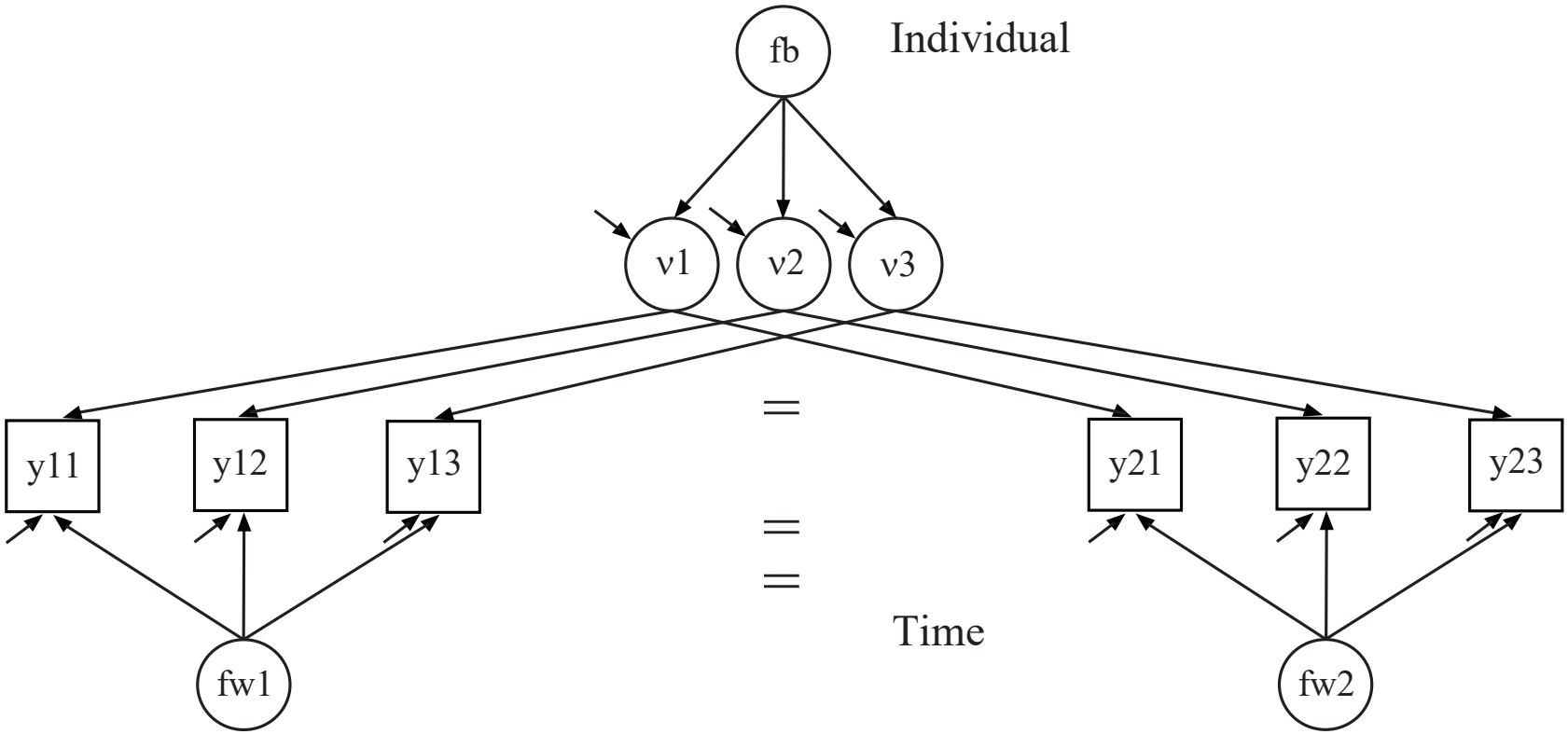
Muthén (1994). Multilevel covariance structure analysis. In J. Hox & I. Kreft (eds.), Multilevel Modeling, a special issue of Sociological Methods & Research

Multilevel Factor Analysis: Two Students Per School



- Cross-sectional: Students within schools
- But the case of time points nested within individuals is also multilevel - so why not apply this model to **longitudinal** data?

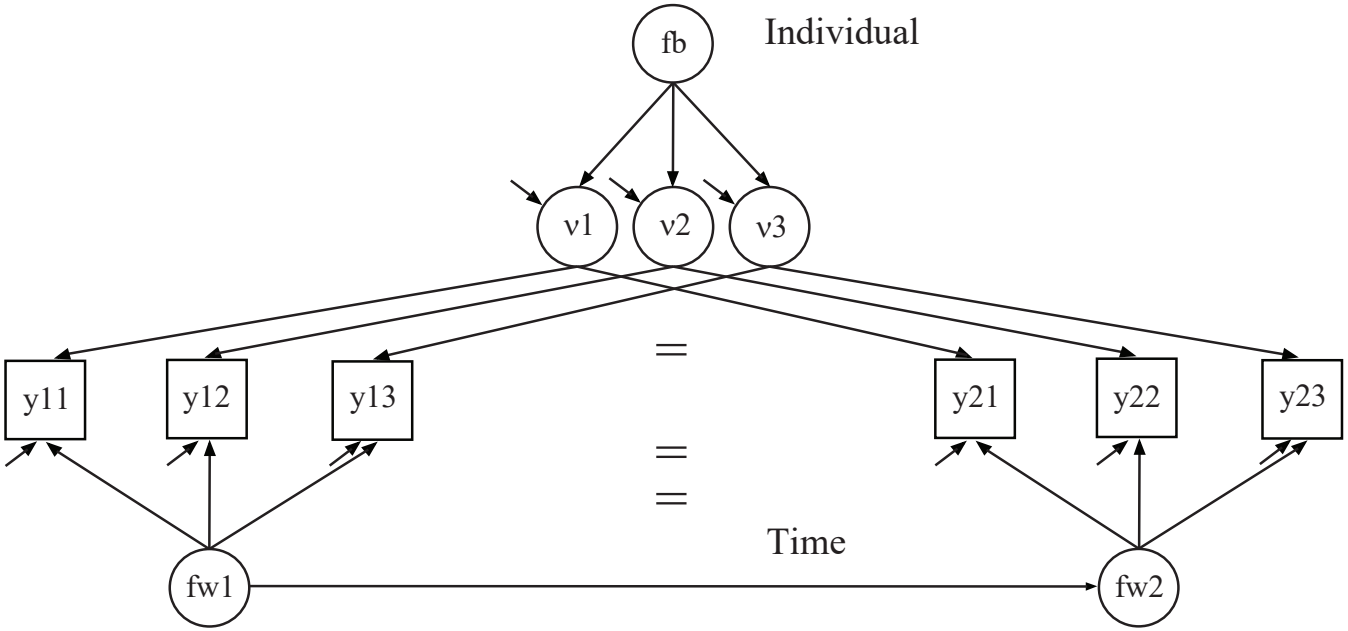
Longitudinal Factor Analysis, T=2



Longitudinal Factor Analysis

- Hamaker et al. (2016). Using a few snapshots to distinguish mountains from waves: Weak factorial invariance in the context of trait-state research. Multivariate Behavioral Research. - "CUTS" (Common and Unique Trait State) model, based on multilevel FA
- Argues that factors obtained from cross-sectional factor analysis are partly determined by between- and partly by within-person covariance structure (Cattell, 1978): "An uninterpretable blend" (R & B, 2002)
- Multilevel-based longitudinal factor analysis allows differences between between-person, trait-like factor structure and within-person, state-like factor structure
- T=2 is sufficient to determine the within-person factor structure. No need for intensive longitudinal data to estimate a factor model for each person (N=1 analysis; Cattell's P-technique)
- Relates to latent state-trait work in Marsh-Grayson (1994) in SEM, Eid (1996) in MPRO, Dumenci-Windle (1996) in MBR, Geiser-Lockhart (2012) in Psych Methods, Geiser et al. (2015) in Behav Res, and Geiser (2020): Longitudinal Structural Equation Modeling with Mplus A Latent State-Trait Perspective. Guilford Press.

Longitudinal Factor Analysis Based on Multilevel Thinking



- Model extended to include auto-regression for the fw factors
- Corresponds to a cross-sectional model where students within schools influence each other
- This multilevel-based longitudinal model is quite different from the ”naive” longitudinal model which uses only the bottom part

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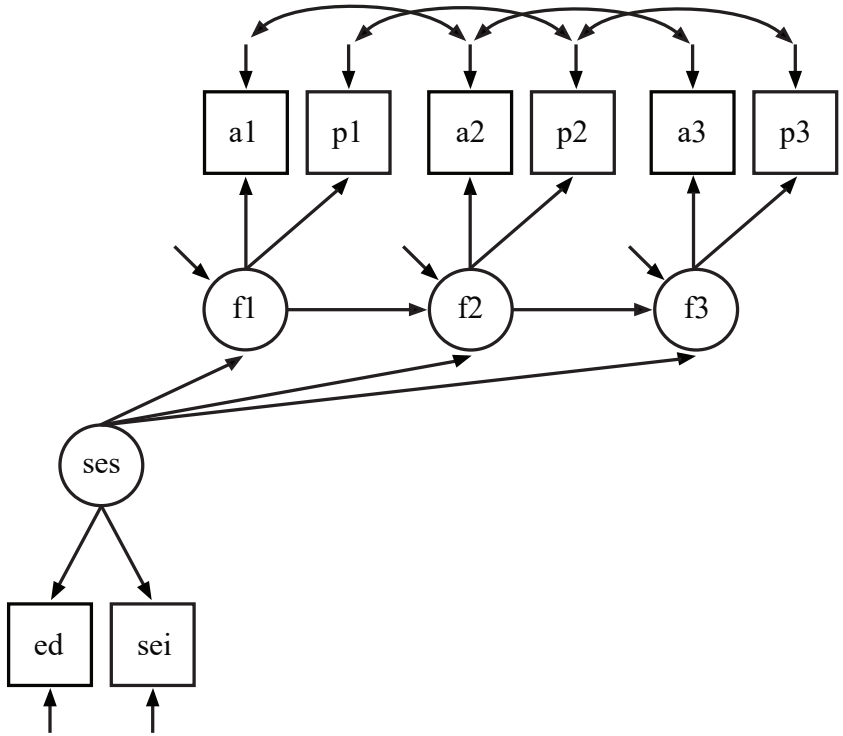

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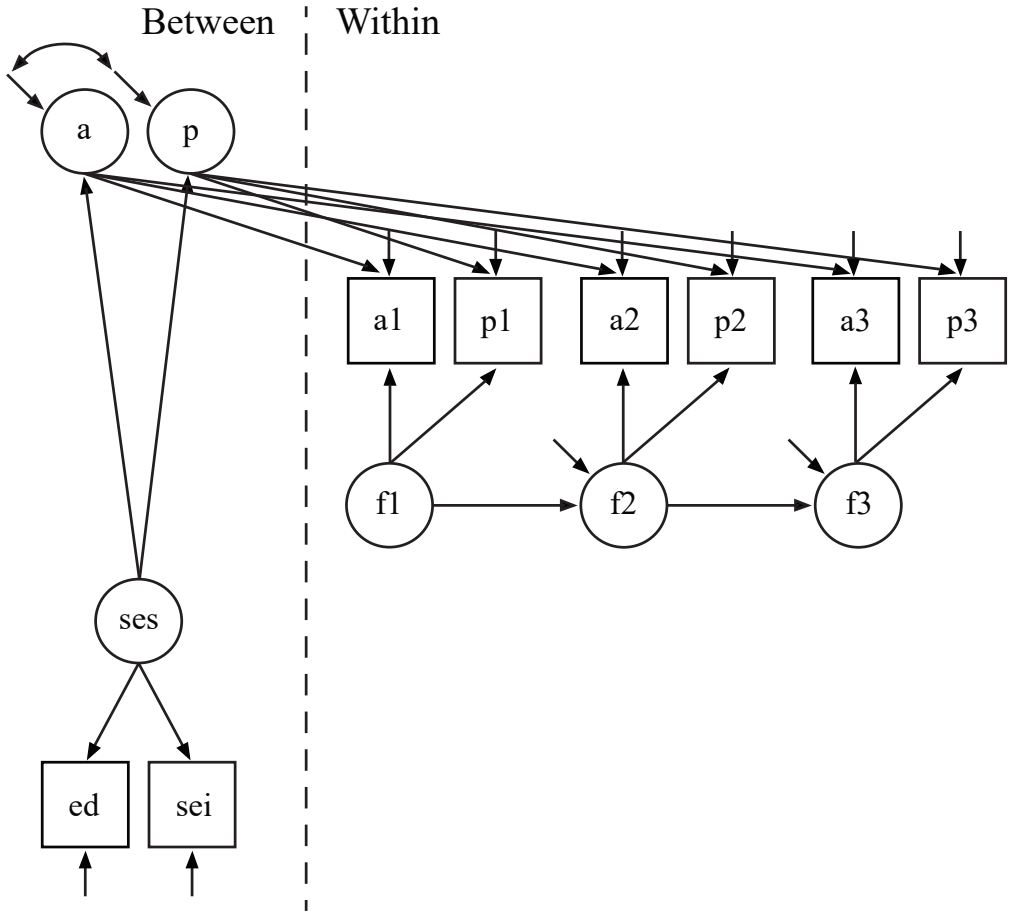
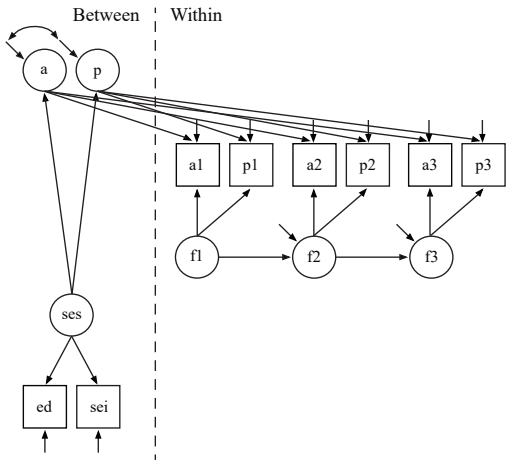
A Random Intercept Version of the "Wheaton et al" Model



Random Intercept Model Features



- Still a single-level, wide model
- Two random intercepts representing ”between-level” variation (stable over time) instead of many correlated residuals
- SES is a ”between-level” variable (does not change over time) and influences only the random intercepts
- The relationships between the factors over time represent within-person dynamics

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- Findings for the New, Random Intercept model:
  - Despite using fewer parameters (25 vs 31), it has better logL, better BIC, and better chi-square, RMSEA, and CFI
  - On Level-2 (Between), SES has a significant negative effect on the random intercepts as expected. This means that SES explains measurement non-invariance across people
  - On level-1 (Within), Positive effect of f1 on f2 (1966 to 1967) but no significant effect of f2 on f3 (1967 to 1971)
- Some across-time equalities can be relaxed for even better fit

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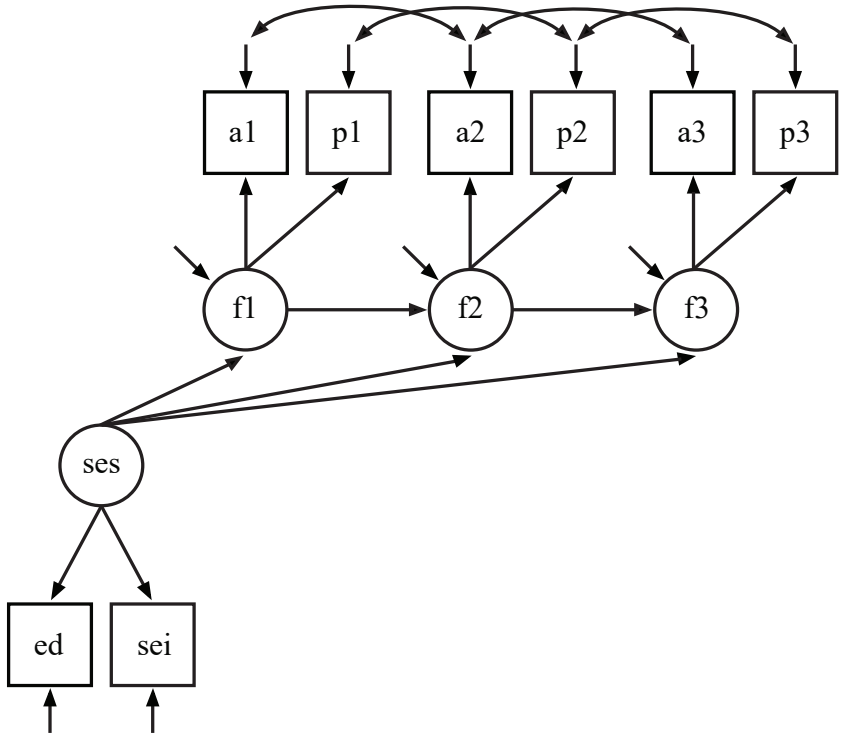
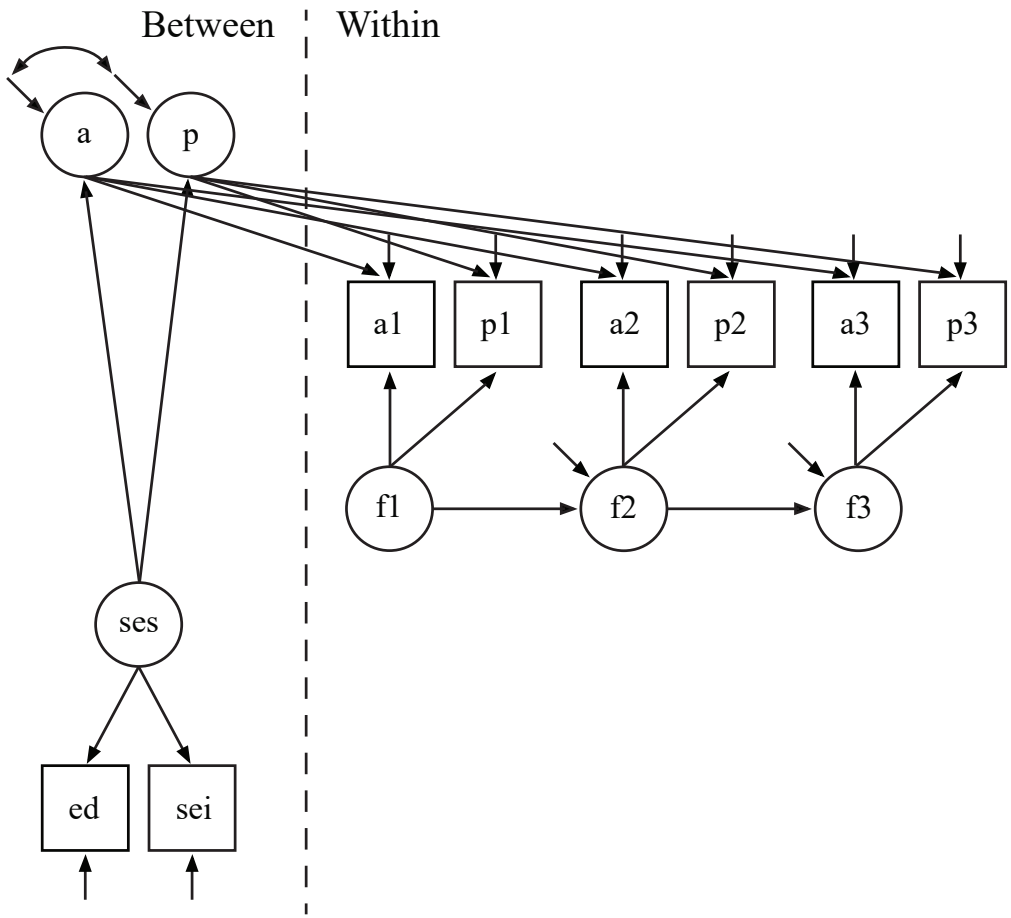
- Output 1: Classic Wheaton et al. model



- Output 2: New Wheaton et al. model



SEM Origins in Madison, Wisconsin

- Karl Jöreskog's LISREL model was presented for the first time at the conference Structural Equation Models in the Social Sciences held at Madison, Wisconsin in November 1970 resulting in an edited book
- Duncan, Otis Dudley, and Arthur S. Goldberger, eds. (1973). Structural Equation Models in the Social Sciences. New York: Seminar Press
- In 1971, after 7 years as a Research Statistician at ETS, Jöreskog returned to Sweden to become Professor of Statistics at Uppsala University until his retirement in 2000
- In 1977, Bengt Muthén received his Ph.D. in Statistics at Uppsala University
- In 1982, Bengt Muthén became professor at UCLA retiring early in 2006 to focus on Mplus
- In 1976, Blair Wheaton received his Ph.D. in Sociology at University of Wisconsin, Madison
- Blair Wheaton is currently Distinguished Professor of Sociology at the University of Toronto

Recent Papers

www.statmodel.com/recentpapers.shtml

- Asparouhov & Muthén (2019). Nesting and equivalence testing for structural equation models. Structural Equation Modeling
- Asparouhov & Muthén (2019). Latent variable centering of predictors and mediators in multilevel and time-series models. Structural Equation Modeling
- Asparouhov & Muthén (2020). Bayesian estimation of single and multilevel models with latent variable interactions. Structural Equation Modeling
- Asparouhov & Muthén (2020). Advances in Bayesian model fit evaluation for structural equation models. Structural Equation Modeling
- Asparouhov & Muthén (2020). Comparison of models for the analysis of intensive longitudinal data. Structural Equation Modeling
- Asparouhov & Muthén (2021). Expanding the Bayesian structural equation, multilevel and mixture models to logit, negative-binomial and nominal variables
- Hamaker & Muthén (2020). The fixed versus random effects debate and how it relates to centering in multilevel modeling. Psychological Methods
- Muthén & Asparouhov (2021). Latent transition analysis with random intercepts (RI-LTA). Forthcoming in Psychological Methods