

TESTING FOR MEASUREMENT INVARIANCE

The MODEL option of the ANALYSIS command is used to automatically test multiple group models and single-group longitudinal models for measurement invariance. For multiple group models, the GROUPING option or the KNOWNCLASS option is used. Measurement invariance testing is available for CFA and ESEM models for continuous variables using the maximum likelihood and Bayes estimators; for censored variables using the weighted least squares and maximum likelihood estimators; for binary and ordered categorical (ordinal) variables using the weighted least squares, maximum likelihood, and Bayes estimators; and for count variables using the maximum likelihood estimator. It is not available for censored-inflated, count-inflated, nominal, continuous-time survival, negative binomial variables, or combinations of variable types. The MODEL command can contain only BY statements for first-order factors. The metric for the factors can be set by fixing a factor loading to one in each group/time point or by fixing the factor variance to one in one group/time point. No partial measurement invariance is allowed. The configural, metric, and scalar models used are described in the next section.

The MODEL option has three settings for testing for measurement invariance: CONFIGURAL, METRIC, and SCALAR. These settings can be used alone to set up a particular model or together to test the models for measurement invariance. Chi-square difference testing is carried out automatically using scaling correction factors for MLM, MLR, and WLSM and using the DIFFTEST option for WLSMV and MLMV. The settings cannot be used together for ESTIMATOR=BAYES and for Monte Carlo analyses. Full analysis results are printed along with a summary of the difference testing. The CONFIGURAL setting produces a model with the same number of factors and the same set of zero factor loadings in all groups/time points. The METRIC setting produces a model where factor loadings are held equal across groups/timepoints. The SCALAR setting produces a model where factor loadings and intercepts/thresholds are held equal across groups/timepoints. For multiple group models, when the factor variance is fixed to one in one group, it is the first group when the GROUPING option is used and the last class when the KNOWNCLASS option is used.

The MODEL option for testing measurement invariance is specified as follows:

MODEL = CONFIGURAL METRIC SCALAR;

which specifies that configural, metric, and scalar models will be estimated and difference testing of the models will be done.

For testing longitudinal measurement invariance, time-specific MODEL commands and an overall MODEL command are used to describe the analysis model. The time-specific MODEL commands are used to specify the factor model to be tested for measurement invariance. A time specific MODEL command must be specified for each time point and the order of the factor indicators must be the same for each time-specific MODEL command. The time-specific MODEL commands are labelled t1 for the first time point, t2 for the second time point, etc.

Following is an example of how to specify the time-specific MODEL commands for a factor measured at three time points. MODEL t1 shows the model at the first time point with factor indicators y11, y12, and y13. MODEL t2 shows the model at the second time point with factor indicators y21, y22, and y23. MODEL t3 shows the model at the third time point with factor indicators y31, y32, and y33.

MODEL t1:

F1 BY y11 y12 y13;

MODEL t2:

F2 BY y21 y22 y23;

MODEL t3:

F3 BY y31 y32 y33;

The overall MODEL command is used to specify relationships across time such as residual covariances, residual auto-regressions, and random intercepts. Following is an example of how to specify residual covariances across time.

MODEL:

y11 WITH y21; y21 WITH y31;

y12 WITH y22; y22 WITH y32;

y13 WITH y23; y23 WITH y33;

The MODEL commands for the measurement models being estimated are useful starting points for testing partial measurement invariance and for adding a structural part to the model. To obtain these, specify MODEL in parentheses following the measurement models in the MODEL option. Following is an example of how this is specified:

MODEL = CONFIGURAL METRIC SCALAR (MODEL);

where MODEL in parentheses following the measurement models specifies that the MODEL commands for the configural, metric, and scalar models will be printed in the output following the results.

MODELS FOR TESTING MEASUREMENT INVARIANCE

Following is a description of the models used by the MODEL option to test for multiple group and longitudinal measurement invariance for various variable types and estimators.

MODELS FOR CONTINUOUS, CENSORED, AND COUNT VARIABLES

Following is a set of models that can be considered for measurement invariance of continuous, censored, and count variables. They are listed from least restrictive to most restrictive. Both

maximum likelihood and Bayes estimators are available for continuous variables. Only the maximum likelihood estimator is available for censored and count variables.

For continuous, censored, and count variables, the configural model has factor loadings, intercepts, and residual variances free across groups/time points and factor means fixed at zero in all groups/time points. If the metric of a factor is set by fixing a factor loading to one, factor variances are free across groups/time points. If the metric of a factor is set by freeing all factor loadings and fixing the factor variance to one, the factor variance is fixed at one in all groups/time points.

The metric model has factor loadings constrained to be equal across groups/time points, intercepts and residual variances free across groups/time points, and factor means fixed at zero in all groups/time points. If the metric of a factor is set by fixing a factor loading to one, factor variances are free across groups/time points. If the metric of a factor is set by freeing all factor loadings within a group/time point and fixing the factor variance to one, the factor variance is fixed at one in one group/time point and is free in the other groups/time points.

The scalar model has factor loadings and intercepts constrained to be equal across groups/time points, residual variances free across groups/time points, and factor means fixed at zero in one group/time point and free in the other groups/time points. If the metric of a factor is set by fixing a factor loading to one, factor variances are free across groups/time points. If the metric of a factor is set by freeing all factor loadings within a group/time point and fixing the factor variance to one, the factor variance is fixed at one in one group/time point and is free in the other groups/time points.

MODELS FOR BINARY VARIABLES

Following is a set of models that can be considered for measurement invariance of binary variables. They are listed from least restrictive to most restrictive. For binary variables and weighted least squares estimation, only the configural and scalar models are considered. The metric model is not identified because scale factors or residual variances are allowed to vary across groups/time points. For binary variables and maximum likelihood estimation, the configural, metric, and scalar models are considered. The metric model is identified because residual variances are implicitly fixed at one in all groups/time points.

WEIGHTED LEAST SQUARES ESTIMATION USING THE DELTA PARAMETERIZATION

For binary variables using weighted least squares estimation and the Delta parameterization, the configural model has factor loadings and thresholds free across groups/time points, scale factors fixed at one in all groups/time points, and factor means fixed at zero in all groups/time points. If the metric of a factor is set by fixing a factor loading to one, factor variances are free across groups/time points. If the metric of a factor is set by freeing all factor loadings and fixing the factor variance to one, the factor variance is fixed at one in all groups/time points.

The scalar model has factor loadings and thresholds constrained to be equal across groups/time points, scale factors fixed at one in one group/time point and free in the other groups/time points, and factor means fixed at zero in one group/time point and free in the other groups/time points. If the metric of a factor is set by fixing a factor loading to one, factor variances are free across groups/time points. If the metric of a factor is set by freeing all factor loadings within a group/time point and fixing the factor variance to one, the factor variance is fixed at one in one group/time point and is free in the other groups/time points.

WEIGHTED LEAST SQUARES ESTIMATION USING THE THETA PARAMETERIZATION

For binary variables using weighted least squares estimation and the Theta parameterization, the configural model has factor loadings and thresholds free across groups/time points, residual variances fixed at one in all groups/time points, and factor means fixed at zero in all groups/time points. If the metric of a factor is set by fixing a factor loading to one, factor variances are free across groups/time points. If the metric of a factor is set by freeing all factor loadings and fixing the factor variance to one, the factor variance is fixed at one in all groups/time points.

The scalar model has factor loadings and thresholds constrained to be equal across groups/time points, residual variances fixed at one in one group/time point and free in the other groups/time points, and factor means fixed at zero in one group/time point and free in the other groups/time points. If the metric of a factor is set by fixing a factor loading to one, factor variances are free across groups/time points. If the metric of a factor is set by freeing all factor loadings within a group/time point and fixing the factor variance to one, the factor variance is fixed at one in one group/time point and is free in the other groups/time points.

MAXIMUM LIKELIHOOD AND BAYES ESTIMATION

For binary variables and maximum likelihood estimation, the configural model has factor loadings and thresholds free across groups/time points and factor means fixed at zero in all groups/time points. If the metric of a factor is set by fixing a factor loading to one, factor variances are free across groups/time points. If the metric of a factor is set by freeing all factor loadings and fixing the factor variance to one, the factor variance is fixed at one in all groups/time points.

The metric model has factor loadings constrained to be equal across groups/time points, thresholds free across groups/time points, and factor means fixed at zero in all groups/time points. If the metric of a factor is set by fixing a factor loading to one, factor variances are free across groups/time points. If the metric of a factor is set by freeing all factor loadings within a group/time point and fixing the factor variance to one, the factor variance is fixed at one in one group/time point and is free in the other groups/time points.

The scalar model has factor loadings and thresholds constrained to be equal across groups/time points and factor means fixed at zero in one group/time point and free in the other groups/time points. If the metric of a factor is set by fixing a factor loading to one, factor variances are free

across groups/time points. If the metric of a factor is set by freeing all factor loadings within a group/time point and fixing the factor variance to one, the factor variance is fixed at one in one group/time point and is free in the other groups/time points.

MODELS FOR ORDERED CATEGORICAL (ORDINAL) VARIABLES

Following is a set of models that can be considered for measurement invariance of ordered categorical (ordinal) variables. They are listed from least restrictive to most restrictive. For ordered categorical (ordinal) variables and weighted least squares estimation, only the configural and scalar models are considered. The metric model is not identified (Wu & Estabrook, 2016). For ordered categorical (ordinal) variables and maximum likelihood estimation, the configural, metric, and scalar models are considered. The metric model is identified because residual variances are implicitly fixed at one in all groups/time points.

WEIGHTED LEAST SQUARES ESTIMATION USING THE DELTA PARAMETERIZATION

For ordered categorical (ordinal) variables using weighted least squares estimation and the Delta parameterization, the configural model has factor loadings and thresholds free across groups/time points, scale factors fixed at one in all groups/time points, and factor means fixed at zero in all groups/time points. If the metric of a factor is set by fixing a factor loading to one, factor variances are free across groups/time points. If the metric of a factor is set by freeing all factor loadings and fixing the factor variance to one, the factor variance is fixed at one in all groups/time points.

The scalar model has factor loadings and thresholds constrained to be equal across groups/time points, scale factors fixed at one in one group/time point and free in the other groups/time points, and factor means fixed at zero in one group/time point and free in the other groups/time points. If the metric of a factor is set by fixing a factor loading to one, factor variances are free across groups/time points. If the metric of a factor is set by freeing all factor loadings within a group/time point and fixing the factor variance to one, the factor variance is fixed at one in one group/time point and is free in the other groups/time points.

WEIGHTED LEAST SQUARES ESTIMATION USING THE THETA PARAMETERIZATION

For ordered categorical (ordinal) variables using weighted least squares estimation and the Theta parameterization, the configural model has factor loadings and thresholds free across groups/time points, residual variances fixed at one in all groups/time points, and factor means fixed at zero in all groups/time points. If the metric of a factor is set by fixing a factor loading to one, factor variances are free across groups/time points. If the metric of a factor is set by freeing all factor loadings and fixing the factor variance to one, the factor variance is fixed at one in all groups/time points.

The scalar model has factor loadings and thresholds constrained to be equal across groups/time points, residual variances fixed at one in one group/time point and free in the other groups/time

points, and factor means fixed at zero in one group/time point and free in the other groups/time points. If the metric of a factor is set by fixing a factor loading to one, factor variances are free across groups/time points. If the metric of a factor is set by freeing all factor loadings within a group/time point and fixing the factor variance to one, the factor variance is fixed at one in one group/time point and is free in the other groups/time points.

MAXIMUM LIKELIHOOD AND BAYES ESTIMATION

For ordered categorical variables and maximum likelihood estimation, the configural model has factor loadings and thresholds free across groups/time points and factor means fixed at zero in all groups/time points. If the metric of a factor is set by fixing a factor loading to one, factor variances are free across groups/time points. If the metric of a factor is set by freeing all factor loadings and fixing the factor variance to one, the factor variance is fixed at one in all groups/time points.

The metric model has factor loadings constrained to be equal across groups/time points, thresholds free across groups/time points, and factor means fixed at zero in all groups/time points. If the metric of a factor is set by fixing a factor loading to one, factor variances are free across groups/time points. If the metric of a factor is set by freeing all factor loadings within a group/time point and fixing the factor variance to one, the factor variance is fixed at one in one group/time point and is free in the other groups/time points.

The scalar model has factor loadings and thresholds constrained to be equal across groups/time points and factor means fixed at zero in one group/time point and free in the other groups/time points. If the metric of a factor is set by fixing a factor loading to one, factor variances are free across groups/time points. If the metric of a factor is set by freeing all factor loadings within a group/time point and fixing the factor variance to one, the factor variance is fixed at one in one group/time point and is free in the other groups/time points.