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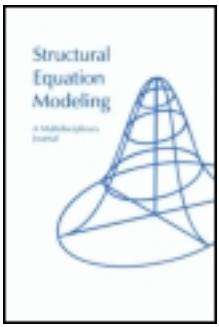

Structural Equation Modeling: A Multidisciplinary Journal
Publication details, including instructions for authors and subscription information:
http://www.tandfonline.com/loi/hsem20

How to Use a Monte Carlo Study to Decide on Sample Size and Determine Power

Linda K. Muthén & Bengt O. Muthén
Published online: 19 Nov 2009.

To cite this article: Linda K. Muthén & Bengt O. Muthén (2002) How to Use a Monte Carlo Study to Decide on Sample Size and Determine Power, Structural Equation Modeling: A Multidisciplinary Journal, 9:4, 599-620, DOI: 10.1207/S15328007SEM0904_8

To link to this article: http://dx.doi.org/10.1207/S15328007SEM0904_8

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TEACHER'S CORNER

How to Use a Monte Carlo Study to Decide on Sample Size and Determine Power

Linda K. Muthén
Muthén & Muthén
Los Angeles, California

Bengt O. Muthén
Graduate School of Education & Information Studies
University of California, Los Angeles

A common question asked by researchers is, "What sample size do I need for my study?" Over the years, several rules of thumb have been proposed. In reality there is no rule of thumb that applies to all situations. The sample size needed for a study depends on many factors, including the size of the model, distribution of the variables, amount of missing data, reliability of the variables, and strength of the relations among the variables. The purpose of this article is to demonstrate how substantive researchers can use a Monte Carlo study to decide on sample size and determine power. Two models are used as examples, a confirmatory factor analysis (CFA) model and a growth model. The analyses are carried out using the M*plus* program (Muthén & Muthén, 1998).

A common question asked by researchers is, "What sample size do I need for my study?" Over the years, several rules of thumb have been proposed, such as 5 to 10 observations per parameter, 50 observations per variable, no less than 100, and so on. In reality, there is no rule of thumb that applies to all situations. The sample size needed for a study depends on many factors, including the size of the model, distri-

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bution of the variables, amount of missing data, reliability of the variables, and strength of the relations among the variables. Although parameter estimates frequently have small bias, standard errors are more sensitive. Standard errors may be overestimated or underestimated depending on the situation. This affects the estimation of confidence intervals also referred to as coverage. If standard errors are overestimated, significant effects may be missed. If they are underestimated, significant effects may be overstated. Another issue that needs to be considered when deciding on sample size is power. A sample may be large enough for unbiased parameter estimates, unbiased standard errors, and good coverage, but it may not be large enough to detect an important effect in the model.

The purpose of this article is to demonstrate how substantive researchers can use a Monte Carlo study to decide on sample size and determine power. Two models are used as examples, a confirmatory factor analysis (CFA) model and a growth model. The analyses are carried out using the M*plus* program (Muthén & Muthén, 1998), which has extensive Monte Carlo facilities. Data generation using M*plus* can include normal data, nonnormal data, missing data, clustering, and mixtures of populations. Analysis models can include any of the models available in M*plus*. Data generation and analysis models do not need to be the same.

This article focuses on parameter estimates, standard errors, coverage, and power assuming correctly specified models. Misspecified models can also be studied in the M*plus* Monte Carlo framework but are not included here. Also, it should be noted that Monte Carlo studies are useful for evaluating the performance of model fit indexes, but this use is not considered in this article.

METHOD

A common use of Monte Carlo studies is for methodological investigations of the performance of statistical estimators under various conditions. In these studies, data are generated and models are estimated, sometimes using more than one estimator. The performance of an estimator is judged by studying parameter estimate bias, standard error bias, and coverage. A less common use of Monte Carlo studies is to decide on sample size and determine power in the design of substantive studies. This use is the focus of this article.

Monte Carlo Study

In Monte Carlo studies, data are generated from a population with hypothesized parameter values. A large number of samples are drawn, and a model is estimated for each sample. Parameter values and standard errors are averaged over the samples. The following criteria are examined: parameter estimate bias, standard error bias, and coverage.

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Several decisions need to be made to carry out a Monte Carlo study. The first is the choice of the model to be studied. This choice is driven by the research question being asked. Once the model is chosen, population values for each parameter of the model must be selected. These values can be obtained from theory or previous research. Estimates from previous studies are often the best estimates available for population values in the Monte Carlo study.

Technical considerations in the Monte Carlo study are the number of samples to be drawn and the seed. The number of samples to be drawn (replications) can be thought of as the sample size for the Monte Carlo study. The number of replications should be increased until stability of the results is achieved. In this study, 10,000 replications are used for each analysis to ensure that stability has been reached. The value of the seed determines the starting point for the random draws of the samples. More than one seed should be used, and the results for the different seeds should be checked for stability.

Models to Be Studied

A CFA model and a growth model were selected for study. These models were chosen because they are often used in practice and are sufficiently different from each other. CFA models are typically cross-sectional and have only a covariance structure. The growth model is longitudinal and has both a mean and a covariance structure.

CFA Model

The CFA model that is studied has two factors, each of which has five continuous factor indicators. The CFA model has 31 free parameters and 24 *df*. A diagram of the CFA model is shown in Figure 1. Data are generated using the following population values. The factor loadings are 0.8. The residual variances of the factor indicators are 0.36. Factor variances are fixed to one to set the metric of the factors. The factor correlation is 0.25. All factor loadings are free. These population values are chosen so that the variances of the factor indicators are one, which makes the parameter values more easily interpretable. The population values result in a reliability of 0.64 for each factor indicator. Reliability is calculated as the ratio of the variance of the factor indicator explained by the factor to the total variance of the factor indicator using the following formula,

$$\lambda^2 \psi / (\lambda^2 \psi + \theta) \quad (1)$$

where λ is the factor loading, ψ is the factor variance, and θ is the residual variance.

The focus of the power investigation in the CFA model is the factor correlation. This parameter is of particular interest because it represents the correlation between the two constructs unattenuated by measurement error. The CFA model can also be

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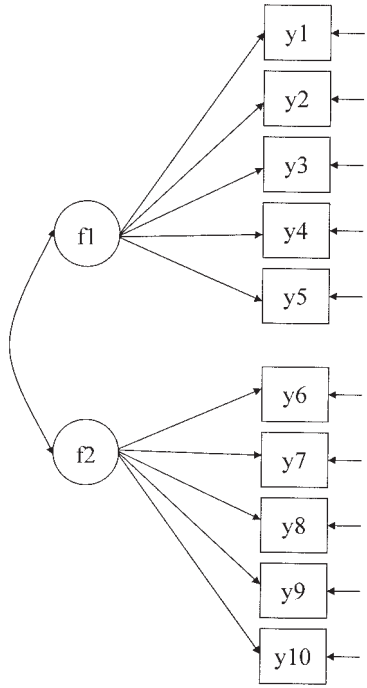

FIGURE 1 CFA model.

thought of as a longitudinal model with two measurement occasions so that the last five indicators are repeated measures of the first five indicators. In this case, the factor correlation can be seen as a measure of stability of the construct over time.

The CFA model is examined under four conditions: (a) normally distributed continuous factor indicators without missing data, (b) normally distributed continuous factor indicators with missing data, (c) nonnormal continuous factor indicators without missing data, and (d) nonnormal continuous factor indicators with missing data.

Missing data. In the analyses with missing data, the data are generated such that all participants have data on $y1$, $y2$, $y3$, $y4$, and $y5$, and 50% of the participants have data on $y6$, $y7$, $y8$, $y9$, and $y10$. The patterns of missing data should be specified to reflect missing data patterns seen in practice. For example, the percentage of missing data can increase in relation to the number of questions in a survey to reflect the likelihood that participants become tired toward the end of a survey and start skipping questions. Or the percentage of missing data can increase over time reflecting the likelihood that people will drop out of a study. If a study is designed such that some participants receive only a subset of the items on a survey or are

measured only at certain ages, this can also be reflected in the generation of data. The way missing data are generated for the CFA model is an example of missing completely at random (Little & Rubin, 1987).

Nonnormal data. In the analyses with nonnormal data, the data are created using a mixture of two normal subpopulations or classes of individuals. Normal data are generated for two classes that have different means and variances for the factor indicators. The combined data are analyzed as though they come from a single population. To maintain a similarity between the CFA models without and with missing data, the parameter values for the factor indicators are chosen so that their reliabilities are 0.64 using Equation 1.

The first step is to generate data for two classes such that the combination of the data from the two classes has the desired skewness and kurtosis. This is done by allowing one of the classes to represent an outlying group of individuals that has different means and variances for the factor indicators. The choice of the proportion of individuals in the two classes also affects skewness and kurtosis. To ensure that the model for the combined data is a correctly specified CFA model, skewness and kurtosis in the factor indicators is achieved by choosing different means and variances for the factors, not by manipulating the means and variances of the factor indicators.

For the CFA model with nonnormal data, Class 1, the outlier class, contains 12% of the participants and Class 2 contains the remaining 88%. Only the factor indicators for the second factor are nonnormal. Therefore, the Class 1 mean for the second factor is chosen to be 15 and the variance 5, as compared to the Class 2 mean and variance of zero and 1. The resulting population univariate skewness for variables *y*6 through *y*10 is 1.2. The resulting population univariate kurtosis for variables *y*6 through *y*10 ranges from 1.5 to 1.6.

The second step is to run the analysis with one replication and a large sample to obtain approximate population values for the one class model. In this article, a sample size of 100,000 is used. Given that factor indicator reliabilities of 0.64 are desired, the third step is to solve for the population residual variances for the factor indicators of the second factor using Equation 1 and use those values as the population values for data generation.

Growth Model

Two growth models are studied. Both are linear growth models with equidistant time points for four continuous outcomes. One has a covariate influencing the intercept and slope growth factors. The growth model without a covariate has 9 free parameters and 5 *df*. The growth model with a covariate has 11 free parameters and 7 *df*. Figure 2 shows the diagram for the growth model with the covariate. Data are generated using the following population values. For the growth model without a covariate, the mean of the intercept growth factor is 0.0

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and the mean of the slope growth factor is 0.2. The variance of the intercept growth factor is 0.5, and the variance of the slope growth factor is 0.1, reflecting a commonly seen variance ratio. The covariance between the intercept and slope growth factors is zero. The residual variances of the continuous outcomes are 0.5. This results in R^2 values of 0.50 for $y1$, 0.55 for $y2$, 0.64 for $y3$, and 0.74 for $y4$ using the following formula,

$$R^2\,(y_t) = (\psi_i + x_t^2\,\psi_s + 2\,x_t\,\psi_{is}) \,/\, (\psi_i + x_t^2\,\psi_s + 2\,x_t\,\psi_{is} + \theta_t) \qquad (2)$$

where ψ_i is the variance of the intercept growth factor, x_t is the time score at time t, ψ_s is the variance of the slope growth factor, ψ_{is} is the covariance between intercept and slope growth factors (set at zero in this case), and θ_t is the residual variance for the outcome at time t. Here the x_t time scores are chosen as 0, 1, 2, and 3.

In the growth model with a covariate, the intercept and slope growth factors are regressed on a dichotomous covariate with a 50/50 split, giving the covariate a mean of 0.5 and a variance of 0.25. This covariate can be thought of as a treatment or sex dummy variable. For the intercept growth factor, the regression coefficient is 0.5. The residual variance for the intercept growth factor is chosen as 0.25. This corresponds to an R^2 value of 0.20 for the intercept growth factor.

The focus of the power investigation in the growth model is the regression coefficient in the regression of the slope growth factor on the covariate. This parameter is selected because across-group differences in development over time are the focus of many longitudinal studies. Regression coefficient values of 0.2 and 0.1 are chosen to study different effect sizes. A regression coefficient of 0.2 has an effect size of 0.63, reflecting a medium effect (Cohen, 1969). A slope of 0.1 has an effect size of 0.32, reflecting a small effect. Here effect size is computed as the ratio of the difference in the slope means for the two values of the covariate divided by the

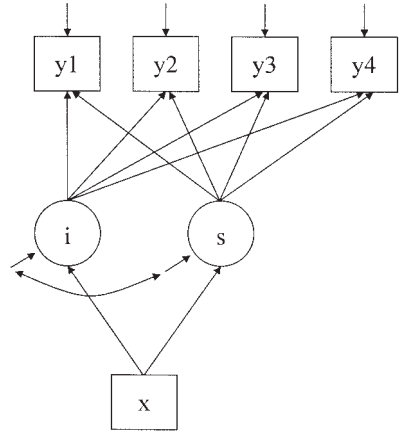

FIGURE 2 Growth model.

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standard deviation of the slope growth factor. The residual variance for the slope growth factor is chosen as 0.09. This corresponds to an R^2 value of 0.10 for the slope growth factor when the regression coefficient is 0.2, and an R^2 of 0.03 when the regression coefficient is 0.1. Values as low as these are commonly seen in the prediction of the slope growth factor.

The growth model is examined under five conditions: (a) normally distributed continuous outcomes without missing data without a covariate, (b) normally distributed continuous outcomes without missing data with a covariate that has a regression coefficient of 0.2 for the slope growth factor, (c) normally distributed continuous outcomes with missing data with a covariate that has a regression coefficient of 0.2 for the slope growth factor, (d) normally distributed continuous outcomes without missing data with a covariate that has a regression coefficient of 0.1 for the slope growth factor, and (e) normally distributed continuous outcomes with missing data with a covariate that has a regression coefficient of 0.1 for the slope growth factor.

Missing data. In the analyses with missing data, the data are generated to reflect an increase in missing data over time due to attrition. For the second through the fourth time-points, the probability of missing data is influenced by the covariate, whereas the first time-point has data missing completely at random. For the covariate value of zero, the first measurement occasion has 12% missing on the outcome, the second has 18% missing, the third has 27% missing, and the fourth has 50% missing. For the covariate value of one, the first measurement occasion has 12% missing on the outcome, the second has 38% missing, the third has 50% missing, and the fourth has 73% missing. The way missing data are generated for the growth model is an example of missing at random (Little & Rubin, 1987).

Model Estimation

Model estimation is carried out in all cases by maximum likelihood under the assumption of normality. For models with nonnormal data, standard errors are computed using a nonnormality robust sandwich estimator. All analyses are done using the M*plus* program. All M*plus* inputs used for this article are included in Appendix A and are available at www.statmodel.com. Complete outputs are also available at this Web site.

Strategy for Deciding on Sample Size

Several criteria are examined to determine sample size. The first criterion is that parameter and standard error biases do not exceed 10% for any parameter in the model. The second criterion is that the standard error bias for the parameter for which power is being assessed does not exceed 5%. The third criterion is that cov-

erage remains between 0.91 and 0.98. Once these three conditions are satisfied, the sample size is chosen to keep power close to 0.80. The value of 0.80 is used because it is a commonly accepted value for sufficient power.

Appendix B shows partial output from the M*plus* analysis for the CFA model with normally distributed continuous factor indicators without missing data. All outputs from the analyses in this article are available at the Web site www.statmodel.com. Following is a description of how the information in the output is used to evaluate the criteria discussed previously.

Parameter bias is evaluated using the information in columns 1 and 2 of the output. The column labeled *starting* gives the population parameter values. The column labeled *average* gives the parameter estimate average over the replications of the Monte Carlo study. For example, the first number in column 2, 0.7963, is the average of the factor loading estimates for *y*1 over 10,000 replications. To determine its bias, subtract the population value of 0.8 from this number and divide it by the population value of 0.8. This results in a bias of –0.005, which is negligible.

Standard error bias is evaluated using the information in columns 3 and 4 of the output. The column labeled *SD* gives the standard deviation of each parameter estimate over the replications of the Monte Carlo study. This is considered to be the population standard error when the number of replications is large. The column labeled *SE Average* gives the average of the estimated standard errors for each parameter estimate over the replications of the Monte Carlo study. Standard error bias is calculated in the same way as parameter estimate bias as described previously.

Coverage is evaluated using the information in column 6 of the output labeled *95% Cover*. It gives the proportion of replications for which the 95% confidence interval contains the true parameter value.

Power is evaluated using the information in column 7 of the output labeled *% Sig Coeff*. This column gives the proportion of replications for which the null hypothesis that a parameter is equal to zero is rejected for each parameter at the .05 level (two-tailed test with a critical value of 1.96). The statistical test is the ratio of the parameter estimate to its standard error, an approximately normally distributed quantity (*z* score) in large samples. For parameters with population values different from zero, this value is an estimate of power, that is, the probability of rejecting the null hypothesis when it is false. For parameters with population values equal to zero, this value is an estimate of Type I error, that is, the probability of rejecting the null hypothesis when it is true.

FINDINGS

CFA Model

The results of the four CFA analyses are found in Table 1. For the simplest CFA model with normally distributed continuous factor indicators and no missing data,

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a sample size of 150 is needed for power of .81 to reject the hypothesis that the factor correlation is zero. By adding the complication of missing data, a sample size of 175 is required for power of .81. Considering the CFA model with nonnormal factor indicators without missing data, a sample size of 265 is needed for a power of .80. Adding the complication of missing data results in the need for a sample size of 315 for power of .81.

Growth Model

The results of the five growth model analyses are found in Table 2. For the simplest growth model without missing data and without a covariate, a sample size of 40 is needed for power of .81 to reject the hypothesis that the mean of the slope growth factor is zero.

By adding a dichotomous covariate with population regression coefficient of 0.2 for the regression of the slope growth factor on the covariate, the sample size requirement to reject the hypothesis that the regression coefficient is zero rises to 150 for a power of .81. By adding the complication of missing data, the sample size requirement increases to 250 for a power of .80. By eliminating the missing data complication and changing the population value of the regression coefficient to 0.1, the sample size requirement is 600 for a power of .80. By adding the complication of missing data to the model with a regression coefficient of 0.1, the sample size requirement rises to 1,025 for a power of .80.

TABLE 1
Sample Size Requirements for the Confirmatory Factor Analysis Model

	No Missing	*Missing*
Normal	150	175
Nonnormal	265	315

TABLE 2
Sample Size Requirements for the Growth Model

	No Missing	*Missing*
No Covariate	40	NA
Regression Coefficient .2	150	250
Regression Coefficient .1	600	1025

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DISCUSSION

This article demonstrated the use of a Monte Carlo study for the purpose of deciding on sample size and determining power. A CFA and a growth model were considered.

For the CFA model, the influences of nonnormality and missing data on sample size requirements were studied. Sample size requirements were found to be influenced more by nonnormality than missing data, at least in this situation where data are missing completely at random. For both normal and nonnormal data, adding the complication of missing data increased the sample size requirement by approximately 18%. Having both nonnormality and missing data approximately doubled the sample size requirement.

For the growth model, the influence of missing data, a covariate, and regression coefficient size on sample size requirements were studied. It was found that the largest impact on the sample size requirement came from including a small regression coefficient for the covariate in the model. Reducing the population value of the regression coefficient from 0.2 to 0.1 increased the sample size requirement approximately four times both with and without missing data. This reflected a change in effect size from medium to small. Including missing data in the model increased the sample size requirement by a factor of approximately 1.7 for both effect sizes.

The results in this article support the fact that sample size requirements depend strongly on many factors. As an example, the sample size requirement of 600 for detecting a small effect size in the growth model is high in contrast to the sample size requirement of 265 for detecting a small factor correlation in the CFA model.

This article demonstrated how substantive researchers can use a Monte Carlo study to decide on sample size and determine power. Two models were considered, and a strategy for deciding on sample size was described. Many variations of the models and strategy described in this article can also be considered. Variations of the CFA model that can be considered are factor cross-loadings or residual covariances. In addition, the number of factors and the number of factor indicators can be varied. Variations of the growth model that can be considered are different choices of the R^2 value for the slope growth factor and the continuous outcomes, residual covariances, free time scores, quadratic models, and piecewise models. In addition, the number of time-points can be varied. Also, if a researcher is interested in power for only one parameter, it is not necessary to have the strict bias requirements for all parameters in the model as suggested in the strategy of this article.

In addition to the models and data complications included in this article, Monte Carlo studies in M*plus* can include investigations of sample size and power in situations with cluster samples (hierarchical data) and mixtures of unobserved subpopulations. This allows studies of sample size and power for multilevel CFA models, three-level growth models, factor mixture models, and growth mixture

models. It is important to investigate the reduction in power due to cluster sampling and due to considering small subpopulations in mixture models.

ACKNOWLEDGMENTS

Preparation of this article was supported by Grant K02 AA 00230 and SBIR Contract N44AA92009, both from the National Institute on Alcohol Abuse and Alcoholism.

This article is based on M*plus* Web Note No. 1, "Using M*plus* Monte Carlo Simulations in Practice: A Note on Assessing Estimation Quality and Power in Latent Variable Models," by Bengt Muthén, and M*plus* Web Note No. 2, "Using M*plus* Monte Carlo Simulations in Practice: A Note on Non-Normal Missing Data in Latent Variable Models," by Bengt Muthén and Tihomir Asparouhov. These notes can be found at www.statmodel.com.

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APPENDIX A

Appendix A contains the M*plus* input files for the nine analyses in the article. Following is a brief description of the M*plus* commands. Details about the input language can be found in the M*plus User's Guide* (Muthén & Muthén, 1998). The TITLE command provides a title for the output. The MONTECARLO command describes the technical details of the Monte Carlo study. The ANALYSIS command provides information about the type of analysis to be performed. The MODEL MONTECARLO command is used to provide the population parameter values to be used in data generation. The MODEL command describes the model to be estimated. The OUTPUT command is used to request extra output.

M*plus* Input File for the CFA Model With Normally Distributed Continuous Factor Indicators Without Missing Data

```
TITLE:          cfa1.inp normal, no missing
MONTECARLO:     NAMES ARE y1-y10;
                NOBSERVATIONS = 150;
```

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```
                        NREPS = 10000;
                        SEED = 53487;
                        NCLASSES = 1;
                        GCLASSES = 1;
                        SAVE = cfa1.sav;
ANALYSIS:               TYPE = MIXTURE;
                        ESTIMATOR = ML;
MODEL MONTECARLO:
                        %OVERALL%
                        f1 BY y1-y5*.8;
                        f2 BY y6-y10*.8;
                        f1@1 f2@1;
                        y1-y10*.36;
                        f1 WITH f2*.25;
MODEL:
                        %OVERALL%
                        f1 BY y1-y5*.8;
                        f2 BY y6-y10*.8;
                        f1@1 f2@1;
                        y1-y10*.36;
                        f1 WITH f2*.25;
OUTPUT:                 TECH9;
```

M*plus* Input File for the CFA Model With Normally Distributed Continuous Factor Indicators With Missing Data

```
TITLE:                  cfa2.inp normal, missing
MONTECARLO:             NAMES ARE y1-y10;
                        NOBSERVATIONS = 175;
                        NREPS = 10000;
                        SEED = 53487;
                        NCLASSES = 1;
                        GCLASSES = 1;
                        PATMISS = y6 (.5) y7 (.5) y8 (.5) y9 (.5) y10 (.5);
                        PATPROB = 1;
                        SAVE = cfa2.sav;
ANALYSIS:               TYPE = MIXTURE MISSING;
                        ESTIMATOR = ML;
MODEL MONTECARLO:
                        %OVERALL%
                        f1 BY y1-y5*.8;
```

```
                    f2 BY y6-y10*.8;
                    f1@1 f2@1;
                    y1-y10*.36;
                    f1 WITH f2*.25;
MODEL:
                    %OVERALL%
                     f1 BY y1-y5*.8;
                    f2 BY y6-y10*.8;
                    f1@1 f2@1;
                    y1-y10*.36;
                    f1 WITH f2*.25;
OUTPUT:             PATTERNS TECH9;
```

M*plus* Input File for the CFA Model With Nonnormal Continuous Factor Indicators Without Missing Data

```
TITLE:              cfa3.inp non-normal, no missing
MONTECARLO:         NAMES ARE y1-y10;
                    NOBSERVATIONS = 265;
                    NREPS = 10000;
                    SEED = 53487;
                    NCLASSES = 1;
                    GCLASSES = 2;
                    SAVE = cfa3.sav;
ANALYSIS:           TYPE = MIXTURE;
                    ESTIMATOR = MLR;
MODEL MONTECARLO:
                    %OVERALL%
                    f1 BY y1-y5*.8;
                    f2 BY y6-y10*.8;
                    f1@1 f2@1;
                    y1-y5*.36 y6-y10*9;
                    f1 WITH f2*.95;
                    [C#1@-2];

                    %C#1%

                    [f1@0 f2@15];
                    f1@1 f2@5;

                    %C#2%
```

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```
                    [f1@0 f2@0];
                    f1@1 f2@1;
MODEL:
                    %OVERALL%
                     f1 BY y1-y5*.8;
                    f2 BY y6-y10*4;
                    f1@1 f2@1;
                    y1-y5*.36 y6-y10*9;
                    f1 WITH f2*.20;
                    [y6-y10*1.42];
OUTPUT:             TECH9;
```

M*plus* Input File for the CFA Model With Nonnormal Continuous Factor Indicators With Missing Data

```
TITLE:              cfa4.inp non-normal, missing
MONTECARLO:         NAMES ARE y1-y10;
                    NOBSERVATIONS = 315;
                    NREPS = 10000;
                    SEED = 53487;
                    NCLASSES = 1;
                    GCLASSES = 2;
                    PATMISS = y6 (.5) y7 (.5) y8 (.5) y9(.5) y10 (.5);
                    PATPROB = 1;
                    SAVE = cfa4.sav;
ANALYSIS:           TYPE = MIXTURE MISSING;
                    ESTIMATOR = MLR;
MODEL MONTECARLO:
                    %OVERALL%
                    f1 BY y1-y5*.8;
                    f2 BY y6-y10*.8;
                    f1@1 f2@1;
                    y1-y5*.36 y6-y10*9;
                    f1 WITH f2*.95;
                    [C#1@-2];

                    %C#1%
```

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```
                    [f1@0 f2@15];
                    f1@1 f2@5;

                    %C#2%

                    [f1@0 f2@0];
                    f1@1 f2@1;
MODEL:
                    %OVERALL%
                     f1 BY y1-y5*.8;
                    f2 BY y6-y10*4;
                    f1@1 f2@1;
                    y1-y5*.36 y6-y10*9;
                    f1 WITH f2*.20;

                    [y6-y10*1.42];
OUTPUT:             PATTERNS TECH9;
```

M*plus* Input File for the Growth Model With Normally Distributed Continuous Outcomes Without Missing Data Without a Covariate

```
TITLE:              growth1.inp normal, no covariate, no missing
MONTECARLO:         NAMES ARE y1-y4;
                    NOBSERVATIONS = 40;
                    NREPS = 10000;
                    SEED = 53487;
                    NCLASSES = 1;
                    GCLASSES = 1;
                    SAVE = growth1.sav;
                    ANALYSIS: TYPE = MIXTURE;
                    ESTIMATOR = ML;
MODEL MONTECARLO:
                    %OVERALL%
                    i BY y1-y4@1;
                    s BY y1@0 y2@1 y3@2 y4@3;
                    [y1-y4@0];
                    [i*0 s*.2];
                    i*.5;
                    s*.1;
```

```
                    i WITH s*0;
                    y1-y4*.5;

                    %C#1%

                    [i*0 s*.2];
MODEL:
                    %OVERALL%
                     i BY y1-y4@1;
                    s BY y1@0 y2@1 y3@2 y4@3;
                    [y1-y4@0];
                    [i*0 s*.2];
                    i*.5;
                    s*.1;
                    i WITH s*0;
                    y1-y4*.5;

                    %C#1%
                    [i*0 s*.2];
OUTPUT:             TECH9;
```

M*plus* Input File for the Growth Model With Normally Distributed Continuous Outcomes Without Missing Data With a Covariate That Has a Regression Coefficient of 0.2 for the Slope Growth Factor

```
TITLE:              growth2.inp normal, covariate, no missing
MONTECARLO:         NAMES ARE y1-y4 x;
                    CUTPOINTS = x (0);
                    NOBSERVATIONS = 150;
                    NREPS = 10000;
                    SEED = 53487;
                    NCLASSES = 1;
                    GCLASSES = 1;
                    SAVE = growth2.sav;
ANALYSIS:           TYPE = MIXTURE;
                    ESTIMATOR = ML;
MODEL MONTECARLO:
                    %OVERALL%
                    [x@0]; x@1;
                    i BY y1-y4@1;
                    s BY y1@0 y2@1 y3@2 y4@3;
```

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```
                    [y1-y4@0];
                    [i*0 s*.2];
                    i*.25;
                    s*.09;
                    i WITH s*0;
                    y1-y4*.5;

                    i ON x*.5;
                    s ON x*.2;

                    %C#1%

                    [i*0 s*.2];
MODEL:
                    %OVERALL%
                     i BY y1-y4@1;
                    s BY y1@0 y2@1 y3@2 y4@3;
                    [y1-y4@0];
                    [i*0 s*.2];
                    i*.25;
                    s*.09;
                    i WITH s*0;
                    y1-y4*.5;
                    i ON x*.5;
                    s ON x*.2;

                    %C#1%
                    [i*0 s*.2];
OUTPUT:             TECH9;
```

M*plus* Input File for the Growth Model With Normally Distributed Continuous Outcomes With Missing Data With a Covariate That Has A Regression Coefficient of 0.2 for the Slope Growth Factor

```
TITLE:              growth3.inp normal, covariate, missing
MONTECARLO:         NAMES ARE y1-y4 x;
                    CUTPOINTS = x (0);
                    NOBSERVATIONS = 250;
                    NREPS = 10000;
                    SEED = 53487;
                    NCLASSES = 1;
```

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```
                    GCLASSES = 1;
                    MISSING = y1-y4;
                    SAVE = growth3.sav;
                    ANALYSIS: TYPE = MIXTURE MISSING;
                    ESTIMATOR = ML;
                    MODEL MISSING:
                    %OVERALL%
                    [y1@-2 y2@-1.5 y3@-1 y4@0];
                    y2-y4 ON x@1;
MODEL MONTECARLO:
                    %OVERALL%
                    [x@0]; x@1;
                    i BY y1-y4@1;
                    s BY y1@0 y2@1 y3@2 y4@3;
                    [y1-y4@0];
                    [i*0 s*.2];
                    i*.25;
                    s*.09;
                    i WITH s*0;
                    y1-y4*.5;

                    i ON x*.5;
                    s ON x*.2;

                    %C#1%

                    [i*0 s*.2];
MODEL:
                    %OVERALL%
                    i BY y1-y4@1;
                    s BY y1@0 y2@1 y3@2 y4@3;
                    [y1-y4@0];
                    [i*0 s*.2];
                    i*.25;
                    s*.09;
                    i WITH s*0;
                    y1-y4*.5;

                    i ON x*.5;
                    s ON x*.2;
```

```
                    %C#1%
                    [i*0 s*.2];
OUTPUT:             TECH9;
```

M*plus* Input File for the Growth Model With Normally Distributed Continuous Outcomes Without Missing Data With a Covariate That Has a Regression Coefficient of 0.1 for the Slope Growth Factor

```
TITLE:              growth4.inp normal, covariate, no missing
MONTECARLO:         NAMES ARE y1-y4 x;
                    CUTPOINTS = x (0);
                    NOBSERVATIONS = 600;
                    NREPS = 10000;
                    SEED = 53487;
                    NCLASSES = 1;
                    GCLASSES = 1;
                    SAVE = growth4.sav;
                    ANALYSIS: TYPE = MIXTURE;
                    ESTIMATOR = ML;
MODEL MONTECARLO:
                    %OVERALL%
                    [x@0]; x@1;
                    i BY y1-y4@1;
                    s BY y1@0 y2@1 y3@2 y4@3;
                    [y1-y4@0];
                    [i*0 s*.2];
                    i*.25;
                    s*.09;
                    i WITH s*0;
                    y1-y4*.5;

                    i ON x*.5;
                    s ON x*.1;

                    %C#1%

                    [i*0 s*.2];
MODEL:
                    %OVERALL%
                    i BY y1-y4@1;
```

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```
                    s BY y1@0 y2@1 y3@2 y4@3;
                    [y1-y4@0];
                    [i*0 s*.2];
                    i*.25;
                    s*.09;
                    i WITH s*0;
                    y1-y4*.5;

                    i ON x*.5;
                    s ON x*.1;

                    %C#1%

                    [i*0 s*.2];
OUTPUT:             TECH9;
```

M*plus* Input File for the Growth Model With Normally Distributed Continuous Outcomes With Missing Data With a Covariate That Has a Regression Coefficient of 0.1 for the Slope Growth Factor

```
TITLE:              growth5.inp normal, covariate, missing
MONTECARLO:         NAMES ARE y1-y4 x;
                    CUTPOINTS = x (0);
                    NOBSERVATIONS = 1025;
                    NREPS = 10000;
                    SEED = 53487;
                    NCLASSES = 1;
                    GCLASSES = 1;
                    MISSING = y1-y4;
                    SAVE = growth5.sav;
ANALYSIS:           TYPE = MIXTURE MISSING;
                    ESTIMATOR = ML;
MODEL MISSING:
                    %OVERALL%
                    [y1@-2 y2@-1.5 y3@-1 y4@0];
                    y2-y4 on x@1;
MODEL MONTECARLO:
                    %OVERALL%
                    [x@0]; x@1;
                    i BY y1-y4@1;
                    s BY y1@0 y2@1 y3@2 y4@3;
```

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```
                    [y1-y4@0];
                    [i*0 s*.2];
                    i*.25;
                    s*.09;
                    i WITH s*0;
                    y1-y4*.5;

                    i ON x*.5;
                    s ON x*.1;

                    %C#1%

                    [i*0 s*.2];
MODEL:
                    %OVERALL%
                    i BY y1-y4@1;
                    s BY y1@0 y2@1 y3@2 y4@3;
                    [y1-y4@0];
                    [i*0 s*.2];
                    i*.25;
                    s*.09;
                    i WITH s*0;
                    y1-y4*.5;

                    i ON x*.5;
                    s ON x*.1;

                    %C#1%

                    [i*0 s*.2];
OUTPUT:             TECH9;
```

APPENDIX B

M*plus* Output Excerpts for the CFA Model With Normally Distributed Continuous Factor Indicators and No Missing Data

Model Results	*Estimates*			*SE*		*95%*	*% Sig*
	Starting	*Average*	*SD*	*Average*	*MSE*	*Cover*	*Coeff*
Class 1							
F1 BY							
Y1	.800	.7963	.0707	.0697	.0707	.949	1.000
Y2	.800	.7981	.0712	.0698	.0712	.942	1.000
Y3	.800	.7962	.0708	.0697	.0708	.946	1.000
Y4	.800	.7975	.0708	.0698	.0708	.944	1.000
Y5	.800	.7971	.0704	.0698	.0704	.947	1.000
F2 BY							
Y6	.800	.7959	.0706	.0697	.0707	.945	1.000
Y7	.800	.7961	.0702	.0697	.0702	.950	1.000
Y8	.800	.7950	.0701	.0697	.0701	.945	1.000
Y9	.800	.7969	.0710	.0698	.0710	.946	1.000
Y10	.800	.7968	.0703	.0698	.0703	.946	1.000
F1 WITH							
F2	.250	.2497	.0864	.0850	.0864	.942	.812
Residual variances							
Y1	.360	.3551	.0523	.0513	.0523	.934	1.000
Y2	.360	.3548	.0523	.0514	.0523	.933	1.000
Y3	.360	.3546	.0529	.0513	.0529	.929	1.000
Y4	.360	.3553	.0525	.0514	.0525	.931	1.000
Y5	.360	.3547	.0526	.0513	.0527	.934	1.000
Y6	.360	.3548	.0516	.0513	.0516	.939	1.000
Y7	.360	.3545	.0524	.0513	.0524	.929	1.000
Y8	.360	.3548	.0520	.0513	.0521	.934	1.000
Y9	.360	.3554	.0524	.0514	.0524	.935	1.000
Y10	.360	.3550	.0525	.0514	.0526	.934	1.000
Variances							
F1	1.000	1.0000	.0000	.0000	.0000	1.000	.000
F2	1.000	1.0000	.0000	.0000	.0000	1.000	.000

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