

LTA Interpretation of Probabilities, Odds, and Odds Ratios Results

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1 LTA Measurement Profiles

In this section we discuss two Mplus output features that are specific to the LTA analysis and may appear to be contradictory at first sight. Here we provide the details for these computations and explain why the outputs are different. The first output is given under the heading **"LATENT CLASS INDICATOR MEANS AND PROBABILITIES FOR EACH LATENT CLASS ..."**, and we will refer to this output as O1. The second output is given under the heading **"RESULTS IN PROBABILITY SCALE ..."**, and we will refer to this output as O2. Both outputs report the conditional probability for a categorical latent class indicator, however, the probability is conditioned on different variables. Generally, in O1 the probability is computed conditional on the latent class variable that the indicator measures, while in O2 the probability is computed conditional on all latent class variables. The situation can become slightly complex when there are covariates in the model that affect these probabilities, which is what this note is intended to clarify. It should be noted that O1 and O2 outputs typically give similar patterns of measurement probabilities and therefore the same interpretation of the latent classes. Both outputs can therefore be used for class re-ordering purposes.

Consider first this simple LTA model where K latent class variables are each measured by P categorical indicators. For simplicity, we assume that the indicators are binary but the same logic applies to the ordered polytomous

case. The LTA model is given by the following equation

$$P(U_{pk} = 0|C_k = l) = \frac{1}{1 + \text{Exp}(-\tau_{pkl})}$$

where $p = 1, \dots, P$, $k = 1, \dots, K$, and $l = 1, \dots, L$, where L is the number of latent classes for each latent class variable. The model typically also includes a lag 1 Markov chain model for the latent class variables

$$P(C_1 = l) = \frac{\text{Exp}(\alpha_l)}{\sum_l \text{Exp}(\alpha_l)}$$

$$P(C_k = l_1|C_{k-1} = l_2) = \frac{\text{Exp}(\alpha_{kl_1l_2})}{\sum_{l_1} \text{Exp}(\alpha_{kl_1l_2})}$$

where $\alpha_L = 0$, $\alpha_{kLl_2} = 0$ for identification purposes.

For this model, U_{ik} measures just one latent class variable C_k , and thus

$$P(U_{ik}|C_1, C_2, \dots, C_K) = P(U_{ik}|C_k),$$

which implies that output O1 contains exactly the same probability values as output O2.

Consider now an expansion of the above model with a covariate X which has a direct effect on one or several of the indicator variables. As an example, consider the model where the first indicator of the first latent class variable U_{11} is affected by X . The model for that variable becomes

$$P(U_{11} = 0|C_1 = l, X) = \frac{1}{1 + \text{Exp}(-\tau_{11l} + \beta_{11l}X)}.$$

Since the variable X is treated as a covariate and the distribution of X is not modelled, it is somewhat tricky to estimate $P(U_{11} = 0|C_1)$. There are two different approaches. The first approach, which is implemented in O1, simply reports

$$P(U_{11} = 0|C_1) \approx P(U_{11} = 0|C_1, X = \bar{X}),$$

where \bar{X} is the mean value of X in the entire sample.

The second approach is implemented in O2 and is slightly more complex. Instead of averaging across the X values and reporting the probability for that average, we compute the probability for all the values in the sample and

then report the average of those probabilities. Furthermore, if we want to compute the most realistic estimate for $P(U_{11} = 0|C_1)$ we should use not the entire sample of X values but only those that are in the corresponding class, i.e., we want to average over the distribution of $[X|C_1]$, which in most cases will be different from the entire sample distribution of X . Since the latent class is unknown and the joint distribution of X and C_1 is not modeled (due to the distribution of X not being modeled), we approximate the distribution $[X|C_1]$ with the sample distribution of X by weighting each observation with its posterior class probability. Finally in O2 we compute the probability conditional not just on one latent class variables but on the entire latent class pattern C_1, \dots, C_K , i.e, $P(U_{11} = 0|C_1, C_2, \dots, C_K)$. To be more specific let

$$p_i(l_1, l_2, \dots, l_K) = P(C_1 = l_1, C_2 = l_2, \dots, C_K = l_K | *)$$

where "*" represents the estimated model and the observed data for the i -observation in the data. We then use the following formula to approximate the estimated probability for U_{11}

$$P(U_{11} = 0|C_1 = l_1, C_2 = l_2, \dots, C_K = l_K) \approx \frac{\sum_{i=1}^N p_i(l_1, l_2, \dots, l_K) P(U_{11} = 0|C_1 = l_1, C_2 = l_2, \dots, C_K = l_K, X = X_i)}{\sum_{i=1}^N p_i(l_1, l_2, \dots, l_K)}$$

where N is the sample size, and X_i is the observed covariate value for the i -th observation.

In the above example, where U_{11} measures just the first latent class variable, this formula simplifies to

$$P(U_{11} = 0|C_1 = l_1, C_2 = l_2, \dots, C_K = l_K) \approx \frac{\sum_{i=1}^N p_i(l_1, l_2, \dots, l_K) P(U_{11} = 0|C_1 = l_1, X = X_i)}{\sum_{i=1}^N p_i(l_1, l_2, \dots, l_K)}.$$

The key thing to understand for the O2 output is that $P(U_{11} = 0|C_1 = l_1, C_2 = l_2, \dots, C_K = l_K)$ will in fact depend on l_2, \dots, l_K even though for any particular value of X it does not. That is because the weighted distribution of X varies over all latent class patterns. Even though the model postulates that

$$P(U_{11} = 0|C_1 = l_1, C_2 = l_2, \dots, C_K = l_K, X) = P(U_{11} = 0|C_1 = l_1, X)$$

the O2 output will in most cases show that

$$P(U_{11} = 0|C_1 = l_1, C_2 = l_2, \dots, C_K = l_K) \neq P(U_{11} = 0|C_1 = l_1)$$

because

$$[X|C_1 = l_1, C_2 = l_2, \dots, C_K = l_K] \neq [X|C_1 = l_1].$$

This can be seen in a practical example in Figure 1 and 2. The probability profile of the indicator is different in the two outputs O1 and O2 and the excerpt from output O2 shows that C_4 affects the probability profile of the indicator even though the indicator is a measurement for C_1 . The probability profile reported in O1 can be approximately viewed as a weighted average across the probability profiles for the full latent class patterns given in O2.

Finally we should note that O1 and O2 are not printed in the Mplus output for all LTA models. For some models (one example is the LTA model without the covariate X), the two outputs are identical and Mplus will produce only O1. For other examples O1 is not computed, and only O2 is reported. The output O1 may not be computed due to latent class indicators that measure more than one latent class variable, or it may not be computed due to model features not directly related to probabilities. In those situations when only the O1 output is computed, the measurement profiles do not depend on the covariates, i.e., the profile results are not conditioned on a particular value of the covariates as in Figure 1. If the measurement profiles depend on the covariates, Mplus computes the profiles conditional on the covariates at their mean values. It is possible, however, to obtain the profiles for any other covariate's values. When the model estimation concludes, within the Mplus editor menu options: Mplus/Calculator, different values for the covariates can be specified and the measurement profiles can be obtained for those values.

Figure 1: LTA output O1

LATENT CLASS INDICATOR MEANS AND PROBABILITIES FOR EACH LATENT CLASS
EVALUATED AT THE COVARIATE SAMPLE MEANS

```
MEAN/PROBABILITY PROFILES FOR C1
      Latent class
      1      2      3
ENDING1
  Category 1  0.058  0.988  0.408
  Category 2  0.942  0.012  0.592
```

Figure 2: LTA output O2

RESULTS IN PROBABILITY SCALE AVERAGED OVER THE COVARIATE SAMPLE DISTRIBUTION FOR THE CLASS PATTERNS

```
Estimate
Latent Class Pattern 1 1 1 1
ENDING1
  Category 1      0.039
  Category 2      0.961
Latent Class Pattern 1 1 1 2
ENDING1
  Category 1      0.150
  Category 2      0.850
Latent Class Pattern 1 1 1 3
ENDING1
  Category 1      0.030
  Category 2      0.970
```

2 Transition Probability Odds

In this section we describe the computation for the LTA output with the heading "TRANSITION PROBABILITY ODDS". The output looks as in Figure 3. This table is computed for every pair of consecutive latent class variable C_t and C_{t+1} . The i -th row and j -th column in this square table contains

$$Od = \frac{P(C_{t+1} = j|C_t = i)}{P(C_{t+1} = i|C_t = i)} \quad (1)$$

where the above probabilities are computed according to the estimated model. These are the odds for C_{t+1} transitioning into another class, as compared to staying in the same class that C_t is at. Note that by definition, all diagonal entries this table are 1. In the round brackets in Figure 3 we compute the 95% confidence limits as

$$[Exp(log(Od) - 1.96SE), Exp(log(Od) + 1.96SE)], \quad (2)$$

where SE is the standard error of $log(Od)$.

If there are covariates X predicting the latent class variables, the above table will depend on the value of the covariate. The odds are then computed as

$$Od = \frac{P(C_{t+1} = j|C_t = i, X = \bar{X})}{P(C_{t+1} = i|C_t = i, X = \bar{X})} \quad (3)$$

where \bar{X} is the sample mean of X . It is possible to obtain the odds table for other covariate values, i.e.,

$$Od = \frac{P(C_{t+1} = j|C_t = i, X = X_0)}{P(C_{t+1} = i|C_t = i, X = X_0)} \quad (4)$$

Figure 3: Transition Probability Odds

TRANSITION PROBABILITY ODDS

TRANSITION TABLE ODDS AND 95% CONFIDENCE INTERVALS FOR C1 TO C2			
1.000 (1.000, 1.000)	1.056 (0.729, 1.528)	0.393 (0.064, 2.429)	0.408 (0.183, 0.910)
0.947 (0.654, 1.371)	1.000 (1.000, 1.000)	0.373 (0.053, 2.616)	0.387 (0.134, 1.113)
2.542 (0.412, 15.696)	2.684 (0.382, 18.840)	1.000 (1.000, 1.000)	1.038 (0.187, 5.766)
2.450 (1.099, 5.461)	2.586 (0.898, 7.446)	0.964 (0.173, 5.356)	1.000 (1.000, 1.000)

Figure 4: Covariate Effect on Transition Probability Odds

COVARIATE EFFECTS ON TRANSITION PROBABILITY ODDS RATIOS

EFFECT OF X

TRANSITION TABLE ODDS RATIO AND 95% CONFIDENCE INTERVALS FOR C1 TO C2		
1.000 (1.000, 1.000)	1.990 (1.085, 3.651)	1.137 (0.409, 3.163)
0.502 (0.274, 0.922)	1.000 (1.000, 1.000)	0.571 (0.275, 1.185)
0.880 (0.316, 2.447)	1.751 (0.844, 3.632)	1.000 (1.000, 1.000)

where X_0 is an arbitrary value. This is available through the Mplus / Calculator menu. The value X_0 is specified in the menu.

When the latent class variables are predicted by a covariate, the effect of the covariate is also presented in terms of the effect it has on the odds ratios as seen in Figure 4. The quantities computed here are the odds ratios

$$OR = \frac{P(C_{t+1} = j | C_t = i), X = X_2)}{P(C_{t+1} = i | C_t = i), X = X_2)} \cdot \frac{P(C_{t+1} = j | C_t = i), X = X_1)}{P(C_{t+1} = i | C_t = i), X = X_1)} \quad (5)$$

where $X_2 = X_1 + 1$, i.e., this represents the relative change in the odds of transitioning when the covariate increase by 1. Due to the logistic nature of the model, this relative change is independent of X_1 . The confidence limits reported in the table in round brackets are computed as

$$[Exp(log(OR) - 1.96SE), Exp(log(OR) + 1.96SE)] \quad (6)$$

where SE is the standard error of $log(OR)$