

Bootstrap P-value Computation

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Mplus computes three types of bootstrap confidence intervals: symmetric, asymmetric and bias-corrected. We describe these methods below and we also discuss how to obtain the corresponding P-values. Mplus currently computes the P-value only for the symmetric method, and that is what can be found in the Mplus output (regardless of which type of confidence interval is requested). The purpose of this note is to describe how one can obtain the P-values corresponding to the other two methods with the information available in Mplus 8.4. This is particularly useful when the confidence interval (asymmetric or bias-corrected) leads to one significance conclusion while the P-value (from the symmetric method) produces another significance conclusion. The contradiction is due to the fact that the P-value reported in the Mplus output always uses the symmetric method, while the confidence interval might use a different method. In such contradictory situations, it is recommended to use the significance conclusion that is based on the confidence interval. If in addition, a P-value is needed to support the result of the asymmetric/bias-corrected confidence interval, the manual method described below for computing such a P-value can be utilized. The discussion here applies to both the maximum-likelihood (ML) estimation as well as the weighted least squares estimation (WLS/WLSMV/WLSM/ULSMV).

1 Symmetric

This confidence interval is computed in Mplus when the option `OUTPUT: CINT;` is specified. Suppose that B bootstrap samples are drawn from the original sample and suppose that x_1, \dots, x_B are the parameter estimates obtained from these bootstrap samples for a particular model parameter x .

The bootstrap SE is computed as follows

$$SE = \sqrt{\frac{B}{B-1} \left(\frac{1}{B} \sum_{i=1}^B x_i^2 - \left(\frac{1}{B} \sum_{i=1}^B x_i \right)^2 \right)} = \sqrt{\frac{1}{B-1} \sum_{i=1}^B (x_i - \bar{x})^2}. \quad (1)$$

If the point estimate for this parameter is x_0 , i.e., this is the estimate obtained from the original sample, then the 95% confidence interval is computed as

$$[x_0 - 1.96SE, x_0 + 1.96SE]. \quad (2)$$

In certain situations, reporting the confidence interval alone is not sufficient as that confidence interval only guarantees significance at the 95% level. It does not carry information about how strong the evidence is for the statistical significance. This can be remedied by also reporting the corresponding P-value. The P-value is the probability that the true parameter is actually 0 (even though the model estimate is not). Very small P-value, such as 0.001, would indicate a substantial statistical evidence that the parameter estimate is significant. The 95% confidence interval does not include the 0 value precisely when the P-value is smaller than 5%.

The P-value corresponding to the confidence interval (2) is computed as

$$2(1 - \Phi^{-1}(x_0/SE)), \quad (3)$$

if x_0 is positive and it is computed as

$$2\Phi^{-1}(x_0/SE), \quad (4)$$

if x_0 is negative, where Φ denotes the standard normal distribution function. These P-values are reported in the Mplus output file.

2 Asymmetric

This confidence interval is computed in Mplus when the option OUTPUT: CINT(BOOT); is specified. Suppose that $x_{(1)} < x_{(2)} < \dots < x_{(B)}$ are the ordered values of the parameter estimate across the bootstrap samples. The 95% confidence interval is computed as

$$[x_{(L)}, x_{(U)}] \quad (5)$$

where L is the nearest integer to $0.025B$ and U is the nearest integer to $0.975B$. The P-value corresponding to this interval can be computed as follows. Suppose that M is such that $x_{(M)} < 0 < x_{(M+1)}$, i.e., M of the values are negative and $B - M$ of the values are positive. If all the values are positive then $M=0$, and if all the values are negative then $M = B$. The two-sided P-value is

$$2 \cdot \min(M/B, 1 - M/B). \quad (6)$$

This P-value is currently not computed in Mplus. The output contains the symmetric interval P-value. It is possible however to manually compute this P-value using the following method.

First, B must be set to at least 1000, i.e., a large number of bootstrap draws are needed. Using the option `PLOT:TYPE IS PLOT3`; we can obtain the bootstrap distribution for every parameter (i.e., the distribution of the values x_i). In the Mplus graphics module select "bootstrap distribution" of the desired parameter using 1000 bins as extra precision is needed. To get the value of M , right-button click on the bootstrap distribution plot and select save plot data. The saved data file has two columns. The first column is the bin value and the second column is the count/frequency (the actual bootstrap values x_i are not currently available). In Excel or Google spreadsheet add the count/frequency values of all the negative bin values. This is the value of M and from there one can compute the P-value using (6).

3 Bias-corrected

This confidence interval is computed in Mplus when the option `OUTPUT:CINT(BCBOOT)`; is specified. Let's assume that x_0 is positive and let N be such that $x_{(N)} < x_0 < x_{(N+1)}$, i.e., N out of the B bootstrap estimates are smaller than the point estimate x_0 . We compute

$$z_0 = \Phi^{-1}(N/B) \quad (7)$$

and this quantity is referred to as the bias-correction. If this quantity is 0, i.e. $N = B/2$, the bias-corrected confidence interval would be identical to the asymmetric confidence interval.

Let L be the nearest integer to

$$B \cdot \Phi(2z_0 - 1.96) \quad (8)$$

and U be the nearest integer to

$$B \cdot \Phi(2z_0 + 1.96) \quad (9)$$

The 95% bias-corrected confidence interval is computed as

$$[x_{(L)}, x_{(U)}]. \quad (10)$$

The corresponding one-sided P-value is computed as

$$\Phi(-2z_0 + \Phi^{-1}(M/B)) = \Phi(-2\Phi^{-1}(N/B) + \Phi^{-1}(M/B)) \quad (11)$$

and to get the two-sided P-value we would multiply by 2. Statistical significance here is given when $M < L$ because that is precisely when the 0 value is below the confidence interval ($M + 1 \leq L$ and therefore $0 < x_{(M+1)} \leq x_{(L)}$). This P-value will be smaller than 0.05 precisely when the confidence interval does not include 0. That is because if $M < L$, then the one-sided P-value

$$\Phi(-2z_0 + \Phi^{-1}(M/B)) < \Phi(-2z_0 + \Phi^{-1}(L/B)) \approx \Phi(-1.96) \approx 0.025. \quad (12)$$

As in the case of the asymmetric confidence interval, Mplus does not compute this P-value but the value can be obtained from the bootstrap distribution plot. Two quantities must be computed here: M and N . The computation of M is exactly as in the previous section, while the computation of N amounts to adding the count/frequency values (second column of the saved file) for all bins (first column of the saved file) that are smaller than the bin value nearest to x_0 . If the point estimate x_0 is negative the one-sided P-value is computed similarly and is equal to

$$1 - \Phi(-2z_0 + \Phi^{-1}(M/B)) = 1 - \Phi(-2\Phi^{-1}(N/B) + \Phi^{-1}(M/B)) \quad (13)$$

while the two-sided P-value would multiply the above quantity by 2.