

# Comparing two-level WLS v.s. single level wide WLS

November 10, 2022

Certain two-level models can be estimated in two different ways: with two-level estimation or with wide format single level estimation. This can be done when the cluster sizes in the two-level model are small, usually less than 5. If the estimation is performed with the maximum-likelihood estimator the likelihood for the two models will be identical and therefore the estimates and their standard errors will be the same. This, however, does not occur with the WLS estimator. In this note we explain the reason for that.

We start with identifying the H1 model for the single level estimation and the two-level estimation. Understanding these models is key, because the WLS estimation obtains the model parameter estimates by minimizing the weighted distance between the estimated model and the H1 model. Let's denote the single level H1 model by H11 and the two-level H1 model by H12. Let's also denote the H0 single level model by H01 and the two-level model by H02. Note that H01 and H02 are equivalent/identical models. They are just estimated differently in Mplus. On the other hand, H11 and H12 are not the same model. The fundamental difference between the two models is the fact that in the H12 model the different observations within the cluster are indistinguishable, while in the H11 model they are distinguishable, i.e., they are not constrained to have the same mean etc. Furthermore, H11 and H12 do not even have the same number of parameters. Consider the case of  $P$  binary variables when there are exactly 2 observations in each cluster. The H11 model will have  $2P$  variables and  $2P(2P + 1)/2 = 2P^2 + P$  parameters. The H12 model will have  $P(P - 1)/2$  within level correlation parameters,  $P(P + 1)/2$  between level covariance parameters and  $P$  means/thresholds, for a total of  $P^2 + P$ , i.e., the H11 model has an additional  $P^2$  parameters.

In single level estimation, the parameter estimates for H01 are obtained by minimizing the distance

$$||H01 - H11||.$$

In two-level estimation, the parameter estimates for H02 are obtained by minimizing the distance

$$||H02 - H12||.$$

Because H11 and H12 are not the same we can't expect the parameter estimates for H01 and H02 to be identical even if the two models are the same.

One possibility to explore differences between H01 and H02, is to compare H11 and H12. H12 is nested within H11 and can be estimated with a single level estimation. This estimation will provide a test of fit for the H12 model. If the model is rejected that means that H02 was fitted to an incorrect model and thus the H01 model should be preferred over the H02 model. Consider as an example the case where the data consists of "male" and "female" dyads. If gender is a predictor for model variables, the H12 model which assumes "males" and "females" have equal means in all variables, will be rejected if estimated as a single level model. This would indicate that estimating a two-level model for such data would be a poor choice, unless gender is used as a predictor. Note also that, if gender is entered as a covariate in the two-level estimation with coefficient fixed to 0 for the H02 model, the coefficient will not be zero in the H12 model. This may result in an acceptable H12 when tested against H11, which ultimately may result in nearly identical model estimates for H01 and H02. Adding gender as a covariate of course will not resolve differences in the variance covariance parameters across the two elements in the dyad.

In summary, for those situations where the cluster size is limited to a small number, single level wide WLS estimation should be preferred as it is likely the safer option. Statistically significant differences between the parameter estimates of H01 and H02 are likely due to an incorrect H12 model. In most practical situations, however, it is not necessary to analyze the data, to know which of the two approaches should be used. If the observations in each cluster are interchangeable the two-level model should be used. If the observations are not interchangeable, it is safer to use a single-level wide format even if the model we want to estimate is a two-level model. This may result, however, in a loss of power. A more parsimonious (and correct) H12 model will likely have more power (than the H11 model) to reject an incorrect

H0 model. This is true also for the ML estimator. Note also that the single-level wide format has additional flexibility. The model can be extended to a model with unequal parameters across the different observations in the clusters, which is not possible in the two-level framework. Finally, a general framework consideration should be given regarding the desired inference. Two-level framework is designed to work with arbitrary cluster sizes. Even if sample sizes in each cluster are small, the target population (population for which inference is desired) doesn't necessarily have small clusters. If inference is desired for the unobserved members of the clusters, the two-level framework is more appropriate.