Standardized Coefficients in Mplus

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1 Overview

We consider the structural equation model defined by the following two equations

$$Y^* = \nu + \Lambda\eta + KX + \varepsilon \tag{1}$$

$$\eta = \alpha + B\eta + \Gamma X + \xi. \tag{2}$$

as in Muthen, B.O. (1998-2004), Appendix 3. Three different sets of standardized coefficients are available in Mplus. The first is set is standardized with respect to the variance of the continuous latent variables η. The second is standardized with respect to the variances of η, Y and X. The exact formulas for these standardizations are available in Muthen, B.O. (1998-2004), Appendix 3. The third set is standardized with respect to the the variances of η and Y but not the variance of X and should be preferred for situations when the X variables are already on a pre-specified metric, for example binary covariates. The third set of standardized coefficients is identical to the second with the exception of the following two formulas

$$K^{***} = D_y^{-1} K \tag{3}$$

$$\Gamma^{***} = \Gamma^* \tag{4}$$

In Mplus version 5, covariance parameters, i.e., off-diagonal elements of residual variance covariance matrices are standardized so that they are on the correlation metric. For example,

$$\psi_{ij}^{**} = \frac{\psi_{ij}}{\sqrt{\psi_{ii}\psi_{jj}}} \tag{5}$$

2 Standard Errors for Standard Coefficients

All standardization formulas are in one of the following 4 types

$$c \cdot x_1^{0.5} \tag{6}$$

$$c \cdot x_1^{-0.5} \tag{7}$$

$$c \cdot x_1^{-0.5} \cdot x_2^{-0.5} \tag{8}$$

$$c \cdot x_1^{0.5} \cdot x_2^{-0.5} \tag{9}$$

where c is the original unstandardized coefficient and x_1 and x_2 are either model parameters, $Var(Y_i)$ or $Var(\eta_j)$. Denote the model parameters by θ. We can obtain standard errors for the expression in (6-9) by the delta method if we have the joint asymptotic variance W for θ, $Var(Y)$, and $Var(\eta)$. The joint asymptotic variance covariance matrix for θ is computed during the model estimation. For example, when we use the maximum likelihood estimator, $Var(\theta)$ is simply the inverse of the information matrix. Using the delta method again W can be computed as

$$W = M^T \cdot Var(\theta) \cdot M \tag{10}$$

where M is the matrix

$$M = \begin{bmatrix} I & \frac{\partial Var(Y)}{\partial \theta} & \frac{\partial Var(\eta)}{\partial \theta} \end{bmatrix} \tag{11}$$

where I is the identity matrix.

When covariates X are present in the structural model then $Var(Y)$ and $Var(\eta)$ depend also on the variance of the covariates Σ_x as well as the parameters θ. In this case we augment the vector of parameter θ to include Σ_x. The correlation between these two sets of parameters can be assumed to be 0. For example for the maximum likelihood estimator the likelihood can be split as two separate parts

$$L(Y|X) + L(X). \tag{12}$$

The first part depends only on the θ parameters and the second only on Σ_x. Thus the information matrix is block diagonal and we can assume 0 correlation between θ and the Σ_x. The asymptotic variance for Σ_x is estimated using the maximum likelihood method, i.e., it is the inverse of the information matrix for the unrestricted mean and variance model.

3 References

Muthen, B.O. (1998-2004). Mplus Technical Appendices. Los Angeles, CA: Muthen & Muthen http://statmodel.com/download/techappen.pdf