Bidirectional association

Tihomir Asparouhov

February 10, 2016

A bidirectional association model is a model where two dependent variables Y_1 and Y_2 are regressed on each other. For this model to be identified typically there is an additional covariate X_1 that is is used as a predictor of only one of the variables. For example the model

$$Y_1 = \alpha_1 + \beta_1 Y_2 + \beta_2 X_1 + \varepsilon_1$$

$$Y_2 = \alpha_2 + \beta_3 Y_1 + \varepsilon_2$$

is one such model. Sometimes this model is also referred to as a non-recursive model, however that term is used more generally for other models as well. The above model is generally identified and easy to estimate when both dependent variables are continuous. The Mplus model specification is simply
Y1 on Y2 X1;
Y2 on Y1;
If one of the two variables however is categorical there are certain complications and clarifications that are necessary. Suppose that Y_1 is categorical. The first model equation then will be

$$Y_1^* = \alpha_1 + \beta_1 Y_2 + \beta_2 X_1 + \varepsilon_1$$

where Y_1^* is the underlying latent variable.

The second equation however can take two very distinct forms. The first model is the latent mediator model where the second equation is now

$$Y_2 = \alpha_2 + \beta_3 Y_1^* + \varepsilon_2.$$

The second model is the observed mediator model

$$Y_2 = \alpha_2 + \beta_3 Y_1 + \varepsilon_2.$$

The difference between the two model is what version of Y_1 is used to predict Y_2. In the latent mediator model we use the underlying latent variable Y_1^* and in the

1

observed mediator model we use the observed categorized variable Y_1. Depending on which estimator is used the specification "Y2 on Y1;" yields one of the above two models. With the Bayes estimator both model specifications are possible and the user can specify MEDIATOR = LATENT; or MEDIATOR=OBSERVED; in the ANALYSIS command. With the WLS, WLSM, WLSMV and ULSMV estimators the latent mediator model is estimated. With the ML, MLR and MLF estimators the observed mediator model is estimated.

When the second equation is combined with the first equation additional complication arise because of the bidirectional association. It turns out that the bidirectional association is not a consistent model when the second equation is the observed mediator model. This is explained in detail in Section 5.7 in G.S. Maddala (1983) Limited Dependent and Qualitative Variables in Econometrics. Thus Mplus will not estimate this bidirectional association model with the ML estimator and Mplus will terminate with the message

*** FATAL ERROR RECIPROCAL INTERACTION PROBLEM.

When the second equation is the latent mediator model the bidirectional association is a consistent model and the model can be estimated in Mplus with the WLSMV, WLSM, WLS, USLMV and Bayes estimators.

The above situation when an inconsistent reciprocal interaction problem occurs in the model can also be found in certain Mixture models where the place of the categorical variable Y_1 is taken by the categorical latent class variable. Mplus will terminate again with the same fatal error message.