Overview of Recent Mplus Developments

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Overview of the Overview

Road map for the study of new papers, presentations, and procedures

Mplus Version 7 was released last September with a Diagrammer and many new features in the following Mplus areas:

- Mediation analysis
- Factor analysis
- Multiple-group analysis
- Longitudinal analysis
- Mixture modeling
- Multilevel modeling

Lots of new possibilities for analysis as well as methods studies.

More on Mplus Version 7

For more information, see Version History at the Mplus web site
www.statmodel.com, including 22 new User's Guide examples

Videos and pdfs from the Mplus Version 7 training at Utrecht
University August 27-29 can be found via the Mplus home page

- LOOP plot for moderated mediation
- Causal effects

LOOP Option

LOOP is used in MODEL CONSTRAINT in conjunction with the PLOT option to create plots of one variable related to another, including a 95% confidence interval. An example:

MODEL: y ON x (p1);
MODEL CONSTRAINT:
PLOT(ypred);
LOOP(age, 20, 50, 1); ! $20 \leq age \leq 50$ with steps of 1
ypred = p1*age;

- Plotting indirect effects with moderated mediation. Preacher, Rucker, Hayes (2007), MBR: Figure 3 - conditional indirect effect as a function of the moderator
- Plotting cross-level interactions in two-level modeling. Bauer & Curran (2005)
- Plotting sensitivity graphs for causal effect mediation modeling. Imai et al. (2010), Psych Methods; Muthén (2011)
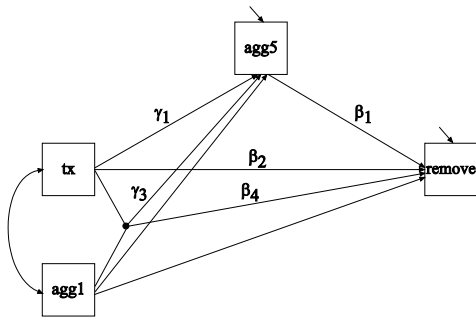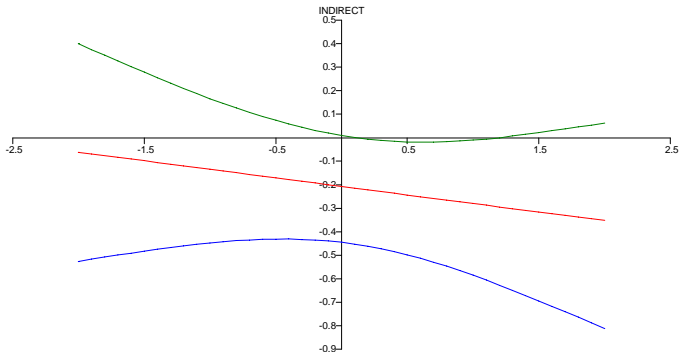
$$remove = \beta_0 + \beta_1 \, agg5 + \beta_2 \, tx + \beta_3 \, agg1 + \beta_4 \, tx \, agg1 + \varepsilon_1, \quad (1)$$

$$agg5 = \gamma_0 + \gamma_1 \, tx + \gamma_2 \, agg1 + \gamma_3 \, tx \, agg1 + \varepsilon_2, \quad (2)$$

$$= \gamma_0 + (\gamma_1 + \gamma_3 \, agg1) \, tx + \gamma_2 \, agg1 + \varepsilon_2. \quad (3)$$

Indirect effect of tx on remove is $\beta_1 \, (\gamma_1 + \gamma_3 \, agg1)$, where agg1 moderates the effect of the treatment. Direct effect: $\beta_2 + \beta_4 \, agg1$.

See Mplus User's Guide ex 3.18

Mediation Analysis:
Causally-Defined Direct and Indirect Effects

Muthén (2011). Applications of Causally Defined Direct and Indirect Effects in Mediation Analysis using SEM in Mplus.

The paper, an appendix with formulas, and Mplus scripts are available at www.statmodel.com under Papers, Mediational Modeling

New ways to estimate mediation effects with categorical and other non-normal mediators and distal outcomes

Causally-Defined Effects

- Causally-defined effects based on counterfactuals and potential outcomes using expectations have been developed by Robins, Greenland, Pearl, VanderWeele, Imai etc
  - Total indirect effect (TIE) and pure indirect effect (PIE)
  - Direct effect (DE)
  - Total effect (TE)
- Same results for SEM with continuous outcomes (but a clearer way of defining them)
- Different results than SEM with for instance categorical DVs
  - Causal effect researchers say that SEM results such as the indirect effect $a \times b$ are wrong
- Most SEM users are not aware of this development

The causal effects TIE, PIE, DE, and TE are expressed in a general way and can be applied to many different settings:

- Continuous mediator, continuous distal outcome (gives the usual SEM formulas)
- Categorical mediator, continuous distal outcome
- Continuous mediator, categorical distal outcome
- Categorical mediator, categorical distal outcome
- Count distal outcome
- Nominal mediator
- Survival distal outcome

Mplus Implementation of Causally-Defined Effects

- The direct and indirect effects can be estimated in Mplus using maximum-likelihood or Bayes
- ML:
    - Standard errors of the direct and indirect causal effects are obtained by the delta method using the Mplus MODEL CONSTRAINT command
    - Bootstrapped standard errors and confidence intervals are also available, taking into account possible non-normality of the effect distributions
- Bayes
    - Bayesian analysis is available in order to describe the possible non-normal posterior distributions
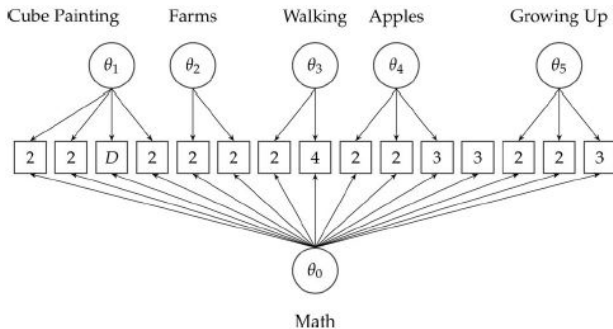
Factor Analysis

- Single-group analysis
    - Parallel analysis to decide on the number of factors
    - Bi-factor EFA rotations
    - Bayesian factor analysis
        - Bayesian EFA
        - Bayesian factor scores (plausible values)
        - Bayesian CFA (BSEM)
    - Two-tier modeling for confirmatory bi-factor analysis
- Analysis of several groups such as cross-cultural comparisons
    - Multiple-group BSEM
    - Two-level analysis with random intercepts and loadings
    - Rotation to invariance simplicity

Bi-Factor Analysis

Bi-factor modeling has a resurgence of popularity used for

- Testlet modeling, e.g. for PISA test items
- Longitudinal modeling with across-time correlation for residuals of the same item

Figure : Bi-Factor Model for PISA Math Items

Bi-Factor EFA Rotations

- Gibbons, & Hedeker, (1992). Full-information item bi-factor analysis. Psychometrika, 57, 423436.
- Cai (2010). A two-tier full-information item factor analysis model with applications. Psychometrika, 75, 581-612.
- Cai, Yang, Hansen (2011). Generalized full-information item bifactor analysis. Psychological Methods, 16, 221-248.
- Reise (2012). The rediscovery of bifactor measurement models. Multivariate Behavioral Research, 47, 667-696.

- Bi-factor EFA (Jennrich & Bentler, 2011, 2012, Psychometrika)
  - Mplus ROTATION = BI-GEOMIN
  - Muthén & Asparouhov (2013). Item response modeling in Mplus: A multi-dimensional, multi-level, and multi-timepoint example. Forthcoming in Linden & Hambleton (2013). Handbook of item response theory: Models, statistical tools, and applications

Bayesian Factor Analysis

- Why Bayes?
- Bayesian EFA
- Bayesian factor scores (plausible values)
- Bayesian CFA (BSEM)

Why Bayes?

- Bayesian analysis firmly established and its use is growing in mainstream statistics
- Much less use of Bayes outside statistics
- Bayesian analysis not sufficiently accessible in other programs
- Bayesian analysis was introduced in Mplus Version 6 and greatly expanded in Version 7: Easy to use
- Bayes provides a broad platform for further Mplus development

Why do we have to learn about Bayes?

- More can be learned about parameter estimates and model fit
- Better small-sample performance, large-sample theory not needed
- Non-informative versus informative priors
- Frequentists can see Bayes with non-informative priors as a computing algorithm to get answers that would be the same as ML if ML could have been done
- Informative priors can better reflect substantive hypotheses
- Analyses can be made less computationally demanding
- New types of models can be analyzed

For a Bayes introduction with further references, see, e.g., Muthén (2010). Bayesian analysis in Mplus: A brief introduction. Technical Report. Version 3.

Bayesian EFA

- Bayesian estimation of exploratory factor analysis implemented in Mplus version 7 for models with continuous and categorical variables
- Asymptotically the Bayes EFA is the same as the ML solution
- Bayes EFA for categorical variable is a full information estimation method without using numerical integration and therefore feasible with any number of factors
- Asparouhov and Muthén (2012). Comparison of computational methods for high dimensional item factor analysis

Bayes Factor Scores Handling

- New improved language for factor scores with Bayesian estimation. The same language as for other estimators
- SAVEDATA: FILE=fs.dat; SAVE=FS(300); FACTORS=factor names; This command specifies that 300 imputations will be used to estimate the factor scores and that plausible value distributions are available for plotting
- Posterior mean, median, confidence intervals, standard error, all imputed values, distribution plot for each factor score for each latent variable for any model estimated with the Bayes estimator
- Bayes factor score advantages: more accurate than ML factor scores in small sample size, Bayes factor score more accurate in secondary analysis such as for example computing correlations between factor

Bayesian CFA (BSEM)

- Regular CFA is too strict, seldom fits well, and overestimates factor correlations
- Bayes CFA (BSEM) is more flexible, using zero-mean-small-variance informative priors to allow for cross-loadings, residual correlations, and direct effects which are not identified in ML

Muthén & Asparouhov (2012). Bayesian SEM: A more flexible representation of substantive theory. Psychological Methods, 17, 313-335. With commentaries and a rejoinder.

Golay, Reverte, Rossier, Favez & Lecerf (2012, November 12). Further insights on the French WISCIV factor structure through Bayesian structural equation modeling. Psychological Assessment. Advance online publication. DOI: 10.1037/a0030676

- Group is fixed mode
    - Multiple-group BSEM with approximate invariance
    - Multiple-group rotational alignment
- Group is random mode
    - Two-level analysis with random item parameters including random loadings

Approximate measurement invariance across groups using zero-mean, small-variance informative priors for the group differences.

Produces "modification indices" by flagging non-invariant items as significantly deviating from average (ML-based MIs not available for categorical items)

Muthén & Asparouhov (2013). BSEM measurement invariance analysis. Web Note 17.

Multiple-Group BSEM Example:
PISA Comparison of 40 Countries

- Items from the PISA (Program for International Student Assessment) survey of 2003
- A total of 9796 students from 40 countries
- Analyzed by Fox (2010). Bayesian Item Response Modeling
- A 40-group, one-factor model for eight mathematics test items
- 2-parameter probit IRT model that accommodates country measurement non-invariance for all difficulty (threshold) and discrimination (loading) parameters as well as country-specific factor means and variances

Multiple-Group BSEM:
Non-Invariance Findings for PISA Items

Table : PISA countries with significant differences relative to the average across countries (prior variance = 0.10)

Item	Loading	Threshold
1	-	2, 12, 18, 22, 28, 39
2	15, 35	29, 38
3	15	23, 34, 35
4	-	12, 27, 40
5	3	7, 37
6	3, 33	5, 18, 25, 27, 37
7	-	9, 24, 27
8	24	-

Figure : Estimated factor means for 40 countries: Comparing BSEM analysis (X axis) with analysis imposing exact invariance (Y axis)
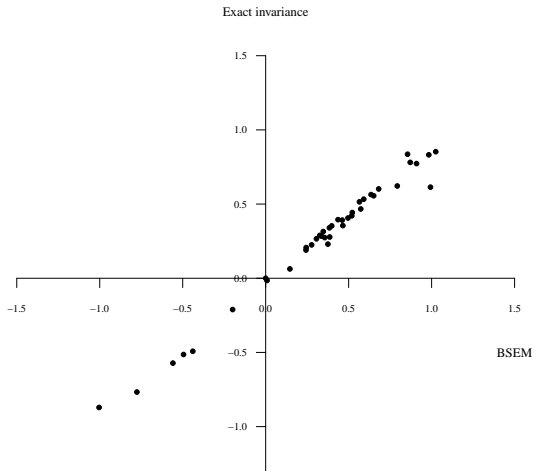
Longitudinal BSEM Example:
Multiple-Indicator Growth Modeling of Aggressive Behavior

Table : Time points with significant differences relative to the average across time points for the aggression example (prior variance $= 0.10$)

Item	Loading	Threshold
stubborn	1, 3, 8	1, 2, 3, 6, 8
breaks rules	1, 2, 8	1, 5, 8
harms others	1, 7, 8	2, 8
breaks things	1, 7	2, 3, 8
yells	1, 3	2, 4, 6, 8
takes property	1, 8	1, 2, 5, 6, 7
fights	2, 8	1, 3, 4
lies	2, 8	-
teases	7, 8	1, 4, 6, 8

Multiple-Group Rotational Alignment

A new method is needed for many groups with many non-invariant parameters

- Step 1: Configural factor analysis - no across-group restrictions
- Step 2: Rotate according to a simplicity criterion that favors few non-invariant measurement parameters
- Step 3: Adjust the factor means and factor variances in line with the rotation

Asparouhov & Muthén (2013). Multiple-group rotational alignment.

Can be combined with BSEM cross-loadings

Forthcoming in Mplus Version 7.1

Two-Level Analysis with Random Item Parameters

- De Jong, Steenkamp & Fox (2007). Relaxing measurement invariance in cross-national consumer research using a hierarchical IRT model. Journal of Consumer Research, 34, 260-278.
- Fox (2010). Bayesian Item Response Modeling. Springer
- Fox & Verhagen (2011). Random item effects modeling for cross-national survey data. In E. Davidov & P. Schmidt, and J. Billiet (Eds.), Cross-cultural Analysis: Methods and Applications
- Asparouhov & Muthén (2012). General random effect latent variable modeling: Random subjects, items, contexts, and parameters
- Bayesian estimation needed because random loadings with ML give rise to numerical integration with many dimensions

- Y_{ijk} - outcome for student i, in country j and item k

$$P(Y_{ijk} = 1) = \Phi(a_{jk}\theta_{ij} + b_{jk})$$

$$a_{jk} \sim N(a_k, \sigma_{a,k}), b_{jk} \sim N(b_k, \sigma_{b,k})$$

This is a 2-parameter probit IRT model where both discrimination (*a*) and difficulty (*b*) vary across country

- The θ ability factor is decomposed as

$$\theta_{ij} = \theta_j + \varepsilon_{ij}$$

- The mean and variance of the ability vary across country
- Model preserves common measurement scale while accommodating measurement non-invariance
- The ability for each country obtained by factor score estimation

- BSEM longitudinal approximate measurement invariance
  - Muthén & Asparouhov (2013). BSEM measurement invariance analysis. Web Note 17
- Intensive longitudinal data (many time points)
  - Individual differences factor analysis (TYPE=TWOLEVEL)
  - Cross-classified longitudinal analysis (TYPE=CROSSCLASSIFIED)

Intensive Longitudinal Data

- Intensive longitudinal data (ILD): More and more longitudinal data are collected with very frequent observations using new tools for data collection such as palm pilots, smartphones etc.
- Ecological Momentary Assessment (EMA) involves repeated sampling of subjects' current behaviors and experiences in real time, in subjects' natural environments
- Experience Sampling Methods (ESM)
- Many time points, small number of subjects

Some Intensive Longitudinal Data Methods References

- Walls & Schafer (2006). Intensive Longitudinal Data. New York: Oxford University Press

- Jahng, Wood & Trull (2008). Analysis of Affective Instability in Ecological Momentary Assessment: Indices Using Successive Difference and Group Comparison via Multilevel Modeling. Psychological Methods, 13, 354-375 (MSSD measure)

- Bolger & Laurenceau (2012). Intensive Longitudinal Methods: An Introduction to Diary and Experience Sampling Research. New York: Guilford Press

- Brose & Ram (2012). Within-Person Factor Analysis. In the new Handbook of Research Methods for Studying Daily Life

Individual Differences Factor Analysis:
Two-Level Analysis with Random Factor Loadings

- Jahng S., Wood, P. K.,& Trull, T. J., (2008). Analysis of Affective Instability in Ecological Momentary Assessment: Indices Using Successive Difference and Group Comparison via Multilevel Modeling. Psychological Methods, 13, 354-375

- An example of the growing amount of EMA data

- 84 outpatient subjects: 46 meeting borderline personality disorder (BPD) and 38 meeting MDD or DYS

- Each individual is measured several times a day for 4 weeks for total of about 100 assessments

- A mood factor for each individual is measured with 21 self-rated continuous items

- The research question is if the BPD group demonstrates more temporal negative mood instability than the MDD/DYS group

Individual Differences Factor Analysis (IDFA)

- This data set is suitable for checking if a measurement instrument is interpreted the same way by different individuals. Some individuals responses may be more correlated for some items, i.e., the correlation matrix could be different for different individuals

- Suppose that one individual always answers item 1 and 2 the same way and a second individual doesn't. We need separate factor analysis models for the two individuals, that is, individual-specific factor loadings

- If the within-level correlation matrix varies across individuals that means that the loadings are individual-specific

- Should factor loadings be individually specific in general? This cannot be determined in cross-sectional studies, only in longitudinal studies with multiple assessments

- IDFA uses TYPE=TWOLEVEL where cluster = individual with many assessments per cluster

- Large across-time variance of the mood factor is considered a core feature of BPD that distinguishes this disorder from other disorders like depressive disorders.

- The individual-specific factor variance is the most important feature in this study

- The individual-specific factor variance is confounded with individual-specific factor loadings

- How to separate the two? Answer: **Using IDFA with a factor model for the random factor loadings**

- Asparouhov & Muthén, B. (2012). General Random Effect Latent Variable Modeling: Random Subjects, Items, Contexts, and Parameters

Current and Future Methods
for Intensive Longitudinal Data

- Dynamic Factor Analysis (see, e.g., Zhang, Hamaker, Nesselroade, 2008 in SEM and references therein)
- Time series analysis (see Hamaker talk at the 4th Dutch Mplus Users' Group meeting via the Mplus home page)
- Mplus latent variable time series developments (see Asparouhov talk at the same meeting)

Cross-Classified Longitudinal Analysis

- Both subject and time are random modes of variation (2 cluster variables)
- Observations nested within time and subject
- A large number of time points can be handled via Bayesian analysis
- A relatively small number of subjects is needed
- Mplus TYPE = CROSSCLASSIFIED
- Allows multiple indicator growth modeling with item parameters varying across time and subject (see UG ex9.27)

Mixture Modeling

- Zero-effect class CACE modeling
- 3-step LCA, LTA, and GMM
- LTA developments

Complier-average causal effect (CACE) modeling is popular:

- Overall treatment effect often insignificant while the effect in the complier class is significant (e.g. JOBS data)
- But the modeling has weaknesses, for example, all compliers are assumed to benefit equally from the treatment
- New solution: Add a complier class with no treatment effect
- Fits JOBS data better and shows a sizeable group who don't benefit, giving a different treatment effect

Sobel & Muthén (2012). Compliance mixture modelling with a zero effect complier class and missing data. Biometrics, 68, 1037-1045

1-step analysis versus 3-step (analyze-classify-analyze) latent class analysis

Critique of 1-Step: Vermunt (2010)
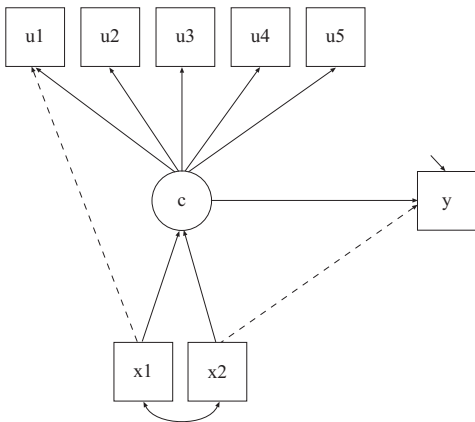
*However, the one-step approach has certain disadvantages. **The first** is that it may sometimes be impractical, especially when the number of potential covariates is large, as will typically be the case in a more exploratory study. Each time that a covariate is added or removed not only the prediction model but also the measurement model needs to be reestimated. **A second** disadvantage is that it introduces additional model building problems, such as whether one should decide about the number of classes in a model with or without covariates. **Third**, the simultaneous approach does not fit with the logic of most applied researchers, who view introducing covariates as a step that comes after the classification model has been built. **Fourth**, it assumes that the classification model is built in the same stage of a study as the model used to predict the class membership, which is not necessarily the case.*

1-Step vs 3-Step: A Hypothetical Genetic Example with Covariates

Substantive question: Should the latent classes be defined by the indicators alone or also by covariates and distal outcomes (antecedents and consequences)?

- Example: Study of genotypes (x variables) influencing phenotypes (y variables)
- Phenotypes may be observed indicators of mental illness such as DSM criteria. The interest is in finding latent classes of subjects and then trying to see if certain genotype variables influence class membership
- Possible objections to 1-step: (1) We want classes determined by DSM alone, not also by genotype covariates; (2) If the genotypes are part of deciding the latent classes, the assessment of the strength of relationship is compromised
- 3-step: Determine the latent classes based on only phenotype information. Then classify subjects. Then relate the classification to the genotypes

1. Estimate the LCA model
2. Determine each subject's most likely class membership
3. Relate the most likely class variable to other variables

The old 3-step approach is problematic: Unless the classification is very good (high entropy), this gives biased estimates and biased standard errors for the relationships with other variables.

The LCA Provides Information About the Classification Quality

Average Latent Class Probabilities for Most Likely
Class Membership (Row) by Latent Class (Column)

	1	2	3
1	0.839	0.066	0.095
2	0.053	0.845	0.102
3	0.125	0.107	0.768

The New 3-Step Approach

- New Method in Mplus Version 7: 3-Step approach correcting for classification error

  1. Estimate the LCA model
  2. Create a nominal most likely class variable *N*
  3. Use a mixture model for *N*, *C* and *X*, where *N* is a *C* indicator with measurement error rates prefixed at the misclassification rate of *N* estimated in the step 1 LCA analysis

- Bolck, Croon, & Hagenaars (2004) Estimating latent structure models with categorical variables: One-step versus three-step estimators. Political Analysis, 12, 3-27.

- Vermunt (2010). Latent Class Modeling with Covariates: Two improved three-step approaches. Political Analysis, 18, 450-469

- Asparouhov & Muthén (2012). Auxiliary variables in mixture modeling: A 3-step approach using Mplus. Mplus Web Note 15.

```
VARIABLE:    NAMES = u1-u5 x;
             CATEGORICAL = u1-u5;
             CLASSES = c(3);
             AUXILIARY = x(R3STEP);
DATA:        FILE = 3step.dat;
ANALYSIS:    TYPE = MIXTURE;
MODEL:       !no model is needed, LCA is default
```

- Latent class models should be subjected to both statistical and substantive checking (Muthén, 2003)
- Substantive checking can be done by relating latent classes to antecedents and consequences (covariates and distal outcomes)
- The 3-step approach is a useful tool for this

3-Step Used for Validation: A PTSD-Dissociation Example with a Validation Variable

- Recent studies of PTSD using CAPS symptoms of both PSTD and dissocation suggest 3 latent classes: Low, high, and high with dissociation
- Substantive question: How does membership in the dissociation subclass relate to validated measures of dissociation such as the Dissociative Experiences Scale (DES)?
- The 3-step method derives latent classes without using the DES and shows how the mean and variance of DES varies across the different latent classes. Do the means exhibit a gradual increase across the 3 classes or a distinct jump when moving to the dissociation class?

u1-u5: CAPS symptom items for PTSD and dissociation
y: Dissociative Experiences Scale (DES)
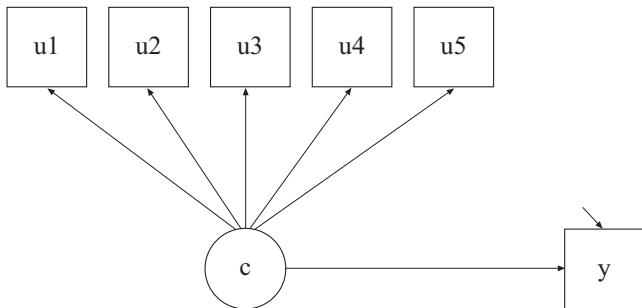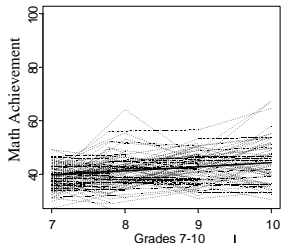
```
VARIABLE:   NAMES = u1-u5 y;
            CATEGORICAL = u1-u5;
            CLASSES = c(3);
            AUXILIARY = y(DU3STEP);
DATA:       FILE = 3step.dat;
ANALYSIS:   TYPE = MIXTURE;
MODEL:      !no model is needed, LCA is default
```
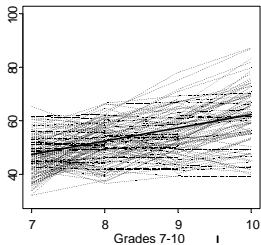
Growth Mixture Modeling: Math Achievement Trajectory Classes and High School Dropout. An Example of Substantive Checking via Predictive Validity
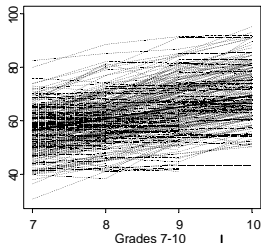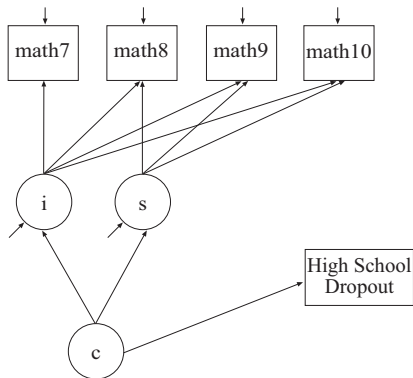

Poor Development: 33%   Moderate Development: 25%   Good Development: 43%

3-Step Analysis Relating Trajectory Classes to the Distal Outcome of High School Dropout

```
VARIABLE:      USEVARIABLES = math7 math8 math9 math10;
               MISSING = ALL(9999);
               CLASSES = c(3);
               AUXILIARY = hsdrop(DE3STEP);
ANALYSIS:
               TYPE = MIXTURE;
               STARTS = 40 8;
               PROCESSORS = 8;
MODEL:
               %OVERALL%
               i s | math7@0 math8@1 math9@2 math10@3;
PLOT:
               TYPE = PLOT3;
               SERIES = math7-math10(s);
```

3-Step Results for the Distal Outcome of High School Dropout

Equality Tests Of Means Across Classes Using The 3-Step
Procedure With 2 Degrees Of Freedom For The Overall Test

HSDROP		
	Means	S.E.
Class 1	0.300	0.020
Class 2	0.043	0.008
Class 3	0.043	0.006
	Chi-Square	P-Value
Overall Test	151.658	0.000
Class 1 vs. 2	116.669	0.000
Class 1 vs. 3	13.306	0.000
Class 2 vs. 3	0.000	0.997

Further 3-Step News

Feingold et al. (2013). New Approaches for Examining Associations with Latent Categorical Variables: Applications to Substance Abuse and Aggression. Psychology of Addictive Behaviors, DOI: 10.1037/a0031487

The Modern Modeling Methods (M3) conference at UConn in May will have a symposium on 3-step mixture methods.

Latent Transition Analysis Developments

New developments in Version 7:

- TECH15 output with conditional class probabilities useful for studying transition probabilities with an observed binary covariate such as treatment/control or a latent class covariate
- LTA transition probability calculator for continuous covariates
- Probability parameterization to simplify input for Mover-Stayer LTA and other models with restrictions on the transition probabilities
- New User's Guide examples
  - 8.13: LTA for two time points with a binary covariate influencing the latent transition probabilities
  - 8.14: LTA for two time points with a continuous covariate influencing the latent transition probabilities
  - 8.15: Mover-stayer LTA for three time points using a probability parameterization

Multilevel Modeling

- Within-cluster multiple-group modeling
- Advantages of Bayesian analysis
- Meta analysis (2-level random)
- 3-level analysis
- Cross-classified analysis
- 3-level and cross-classified multiple imputation
- Applications to Item Response Theory modeling

Within-cluster multiple-group modeling

Asparouhov & Muthén (2012). Multiple group multilevel analysis.
Web note 16.

Two-level SEM with multiple groups:

- Group as level-2 variable (e.g. public vs private schools) is
  straightforward - independent groups
- Group as level-1 variable (e.g. gender within schools) needs
  special treatment - groups are correlated due to common level-2
  units
- New approach:
    - Mixture solution - level-1 groups are latent classes
    - Allows different level-1 groups to have different cluster effects
      (teacher effect different for males and females)

Advantages of Bayesian Multilevel Analysis

With carefully chosen priors, Bayes allows a smaller number of level 2 or level 3 units. See, e.g.,

Muthén (2010). Bayesian analysis in Mplus: A brief introduction. Technical Report. Version 3.

Meta Analysis

- Meta-analysis pools information from several studies designed to address the same scientific question
- Data frequently are in the form of summary statistics from each study, such as effect measures, means, (log) odds ratios, relative risks, z-transformed correlations, and the associated sampling variances
- A normal model for the summary statistic y_j in study j assumes
  $y_j \sim N(\theta_j, \sigma_j^2)$,
  where σ_j^2 is assumed known, estimated from data
- A random-effects model specifies
  $\theta_j \sim N(\mu, \tau^2)$
- A Bayesian model adds priors such as
  $\mu \sim N(0, 1000)$, $\tau^2 \sim U(0, 1000)$

- Heterogeneous variances can be handled by random slopes as in Mplus Web Note # 3. See also Users Guide ex 3.9 (random coefficient regression)

- A similar approach is used in
  Cheung (2008). A model for integrating fixed-, random-, and mixed-effects meta-analyses into structural equation modeling. Psychological Methods, 13, 182-202
  $y_j = \theta_j + \varepsilon_j; \varepsilon_j \sim N(0, \sigma_j^2)$
  Dividing by σ_j,
  $y_j^* = 0 + \theta_j * x_j + \varepsilon_j; \varepsilon_j \sim N(0, 1)$
  where $y_j^* = y_j / \sigma_j, x_j = 1/\sigma_j$ and θ_j is a random slope $\sim N(\mu, \tau^2)$

- Random slope approach using ML or Bayes
  TYPE=TWOLEVEL RANDOM

- See video and handout for Topic 9 of 6/1/11, slides 150-169

Continuous outcomes: ML and Bayesian estimation

Categorical outcomes: Bayesian estimation (Bayes uses probit)

Count and nominal outcomes: Not yet available

Types Of Observed Variables In 3-Level Analysis

Each Y variable is decomposed as

$$Y_{ijk} = Y_{1ijk} + Y_{2jk} + Y_{3k},$$

where Y_{1ijk}, Y_{2jk}, and Y_{3k} are components of Y_{ijk} on levels 1, 2, and 3. Here, Y_{2jk}, and Y_{3k} may be seen as random intercepts on respective levels, and Y_{1ijk} as a residual

- Some variables may not have variation over all levels. To avoid variances that are near zero which cause convergence problems specify/restrict the variation level
- WITHIN=Y, has variation on level 1, so Y_{2jk} and Y_{3k} are not in the model
- WITHIN=(level2) Y, has variation on level 1 and level 2
- WITHIN=(level3) Y, has variation on level 1 and level 3
- BETWEEN= Y, has variation on level 2 and level 3
- BETWEEN=(level2) Y, has variation on level 2
- BETWEEN=(level3) Y, has variation on level 3

Types Of Random Slopes In 3-Level Analysis

- Type 1: Defined on the level 1
  %WITHIN%
  s | y ON x;
  The random slope *s* has variance on level 2 and level 3
- Type 2: Defined on the level 2
  %BETWEEN level2%
  s | y ON x;
  The random slope *s* has variance on level 3 only
- The dependent variable can be an observed Y or a factor. The covariate X should be specified as WITHIN= for type 1 or BETWEEN=(level2) for type 2, i.e., no variation beyond the level it is used at

Level-2 randomized trial (Hox artifical data)
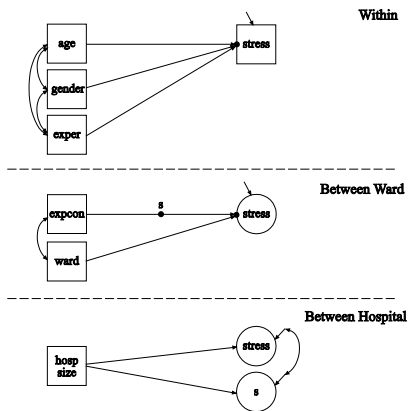
Students are cross-classified by school and neighbourhood at level 2.
An example with 33 students:

	School 1	School 2	School 3	School 4
Neighbourhood 1	XXXX	XX	X	X
Neighbourhood 2	X	XXXXX	XXX	XX
Neighbourhood 3	XX	XX	XXXX	XXXXXX

Source: Fielding & Goldstein (2006). Cross-classified and multiple
membership structures in multilevel models: An introduction and
review. Research Report RR 791, University of Birmingham.

Types of Cross-Classified Analyses in Mplus

- Regression analysis
- Path analysis (both subject and context are random modes)
  - Gonzalez, de Boeck, & Tuerlinckx (2008). A double-structure structural equation model for three-mode data. Psychological Methods, 13, 337-353
- SEM
- Longitudinal analysis (both subject and time are random modes)
- Random items (both subject and item are random modes)
- General idea: Two random modes

3-Level and Cross-Classified Multiple Imputation

New Multiple Imputation Methods

- Multiple imputations for three-level and cross-classified data
- Continuous and categorical variables
- H0 imputations. Estimate a three-level or cross-classified model with the Bayes estimator. Not available as H1 imputation where the imputation model is setup as unrestricted model.
- The imputation model can be an unrestricted model or a restricted model. Restricted models will be easier to estimate especially when the number of clustering units is not large
- In the input file simply add the DATA IMPUTATION command

Applications to Item Response Theory Modeling

Random items, Generalizability Theory

- Items are random samples from a population of items
- The same or different items may be administered to individuals
- Suited for computer generated items and adaptive testing
- 2-parameter IRT model

$$P(Y_{ij} = 1) = \Phi(a_j\theta_i + b_j)$$

- $a_j \sim N(a, \sigma_a)$, $b_j \sim N(b, \sigma_b)$: random discrimination and difficulty parameters
- The ability parameter is $\theta_i \sim N(0, 1)$
- Cross-classified model. Nested within items and individuals. 1 or 0 observation in each cross-classified cell
- Interaction of two latent variables: a_j and θ_i
- The model has only 4 parameters - much more parsimonious than regular IRT models

The Modern Modeling Methods (M3) conference at UConn in May will have 2 complementary 1-day workshops on multilevel latent variable modeling in Mplus:

- Pre-conference workshop by Kris Preacher
- Post-conference workshop by Bengt Muthén

For more information, see the home page of www.statmodel.com.

Recent Mplus Papers

- Asparouhov & Muthén (2010). Computing the strictly positive Satorra-Bentler chi-square test in Mplus. Mplus Web Notes: No. 12
- Muthén (2010). Bayesian analysis in Mplus: A brief introduction. Technical Report. Version 3
- Muthén (2011). Applications of Causally Defined Direct and Indirect Effects in Mediation Analysis using SEM in Mplus.
- Muthén & Asparouhov (2011). LTA in Mplus: Transition probabilities influenced by covariates. Mplus Web Notes: No. 13
- Asparouhov & Muthén (2012). Using Mplus TECH11 and TECH14 to test the number of latent classes. Mplus Web Notes: No. 14
- Asparouhov & Muthén (2012). Comparison of computational methods for high dimensional item factor analysis
- Asparouhov & Muthén (2012). General random effect latent variable modeling: Random subjects, items, contexts, and parameters
- Asparouhov & Muthén (2012). Auxiliary variables in mixture modeling: A 3-step approach using Mplus. Mplus Web Note 15
- Asparouhov & Muthén (2012). Multiple group multilevel analysis. Web Note 16
- Muthén & Asparouhov (2012). Bayesian SEM: A more flexible representation of substantive theory. Psychological Methods, 17, 313-335
- Sobel & Muthén (2012). Compliance mixture modelling with a zero effect complier class and missing data. Biometrics, 68, 1037-1045
- Asparouhov & Muthén (2013). Multiple-group rotational alignment. In preparation
- Muthén & Asparouhov (2013). Item response modeling in Mplus: A multi-dimensional, multi-level, and multi-timepoint example
- Muthén & Asparouhov (2013). BSEM measurement invariance analysis. Web Note 17